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Descriptive capacity and indicative value of territorial variables in ecological cartography

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Abstract

A method is described for selecting different sets of indicator variables from sectors of an ecological hierarchical map. The indicator selection is obtained by analyzing the spatial scale on which each variable attains its maximum indicator capacity. This capacity for prediction is examined by preparing mutual information spectra of each variable using different scales of detail, that can also be viewed as hierarchical levels in a classification.

Introduction

An ecological map should be the spatial expression of a set of relationships between biological and environmental variables. The spatial variability of these relationships may refer to both structure and function of ecological systems. Structure can be mapped by means of the analysis of the spatial concurrence or the correlation between ecological variables, while function may be mapped by means of parameters such as production or turnover rate.

Multivariate analysis, normally employing ordination and classification techniques, has allowed some authors to create different types of maps which show the global variability of the territory in an integrated form (Allaire ***et al.*** **1973;** Bunce ***et al.*** **1975;** Frondorf ***et al.*** **1978;** Bunce ***et al.*** **1981;** Gallopín **1982;** De Pablo and Pineda, **1985).** These maps may include sectors and subsectors obtained in a hierarchical form, so that the structural aspects being mapped are represented with different levels of detail. The application of concepts contained in information theory (Shannon and Weaver **1949;** Quastler **1953;** Margalef **1957;** Abramson **1963;** Godron **1966;** Legendre and Legendre **1977;** Pineda ***et al.*** **1981)** can also constitute a methodology for producing maps which respond to different objectives of territorial management (Cartan **1975;** Phipps **1981;** Moss **1985;** De Pablo ***et al.*** **1987).**

The problems created by ecological maps include: (i) identification of cartographic units that accurately reflect the structure of the territory and (ii) once this accurate map is obtained, its utility for environmental planning. The former problem has been treated by De Pablo ***et al.*** **(1987).** Solution of the latter problem is equivalent to extracting variables from this map. The sets of indicator variables selected should be those which are of greatest importance to a concrete objective, and could include, for example, lithology and geo-tectonic variables for the planning of large-scale public works; vegetation, fauna and agrarian uses for nature conservation; climate for biogeography. This selection permits the use of the same accurate ecological map in a variety of environmental planning objectives.

The detection of variables which characteristically appear in each of the cartographic sectors is equivalent to comparing the spatial distribution of the states of each variable with the sectorization present on the map. Analysis of the coincidence between variable distribution and cartographic sectors is equal to the study of the relation between the indicator and its subject (Viktorov *et al.* 1962; Margalef 1969; Nicolas *et* al. 1979; Bernaldez 1981).

Hierarchical dimensions may also be added to ecological maps by considering their division into sectors and subsectors. The most satisfactory indicator variables for each hierarchical level or spatial scale can then be selected.

This paper presents a method to determine the spatial scale at which each variable attains its maximum indicator capabilities. This allows the selection of the best set of indicators for a spatial scale and, hence, the accurate use of the ecological map for environmental planning purposes. The relation between territorial variables and cartographic sectors on a hierarchical ecological map is studied employing concepts of information theory.

Analysis of entropy between a variable and a map

The information that a variable provides for the characterization of a territory can be taken to be the degree of uncertainty about the presence or absence of its states in the mapped sectors of the territory. When uncertainty is maximum the states of a variable may be equally present or absent in all the sectors. In other words, there would be no preference or exclusion between the states of a variable and these mapped sectors. If, on the other hand, uncertainty is minimal, the states of the variable would be characteristically present or absent in the sectors.

The uncertainty of both finding a given state of the variable at a set point in the territory and, simultaneously, being able to determine its pertenance to a sector, represents the total entropy of the system formed by the variable and the map. This entropy reaches its maximum value when uncertainty is maximum, that is, when the states of the variable and the sectors of the map are unrelated. Its minimum value is obtained when close correspondence exists between both.

Following Shannon and Weaver (1949); Abramson (1963) and De Pablo *et al.* (1987), it is possible to explain the meaning of the total entropy or H(V.S):

$$H(V.S) = H(V) + H(S|V) = H(S) + H(V|S) \quad (1)$$

H(V) represents the entropy of the variable, which is the uncertainty concerning the presence of the different states of the variable in a given observation. Its value is maximum when these states are equally probable.

H(S|V) is the entropy of the sector conditioned by the variable. It represents the uncertainty as to whether or not an observation belongs to a sector when it is already known which state the variable takes. Its value is minimum when each of the states situates the observation in a given sector and maximum when this interdependence does not exist.

H(S) represents the entropy of the sectorization or map. It measures the uncertainty about the presence of an observation in a sector of the map. Its value is maximum when all the sectors are equally probable.

H(V|S) is the entropy of the variable conditioned by the sector. It represents the uncertainty about the state which a variable takes at a set point, once it is clear to which sector it belongs. Its minimum value is reached when the location of a point in a given sector determines the specific state a variable will take in this sector.

The values of H(S|V) and H(V|S) are closely related to the spatial correspondence that could exist between the distribution of the states of the variable and that of the sectors. When these coincide exactly both parameters have a minimum value. If, on the contrary, the distribution of both are completely independent, then the parameters achieve their maximum values (the procedure for calculating these values can be found in Appendix 1).

The relations between these parameters can be used to identify the tendency of a variable to be characteristically situated in specific sectors and the capacity of a map to 'isolate' defined states of the variable in its sectors. These relations can be synthesized in the parameter I(V.S) (Abramson, 1963) which measures the mutual information of the variable and the sectorization:

$$I(V.S) = H(V) - H(V|S) \quad (2)$$

$$= H(S) - H(S|V) \quad (3)$$

$$= H(V) + H(S) - H(V.S) \quad (4)$$

I(V.S) represents the information on the spatial distribution of the states of a variable supplied by a given map and, simultaneously, the information on the distribution of the sectors on the map supplied by this variable. The I(V.S) value depends on the relation which H(S|V) and H(V|S) maintain with H(S) and H(V) respectively. These latter values represent the maximum that the former can achieve in any studied case.

For a given variable and sectorization, I(V.S) achieves its maximum value when uncertainty concerning the presence or absence of the states of a variable in different sectors is minimal; in other words, when each state of the variable unequivocally corresponds to a defined set of sectors and vice versa. If I(V.S) = H(V), and hence H(V|S) = 0, the variable achieves its maximum possibilities of predicting the sectors. All the possible mutual information is already shared. Therefore, the variable could be regarded as a satisfactory indicator of the spatial distribution of sectors. Equally, if I(V.S) = H(S), and hence H(S|V) = 0, the map achieves its highest possibilities of identifying the spatial distribution of the states of the variable.

The ratio C = I(V.S)/H(V) assesses the degree to which a variable is an indicator of the set of sectors contained in a map. If **C** = 1, then I(V.S) = H(V) and the variable's states and map sectors are closely related. Therefore the variable is a good indicator of the sectors' spatial distribution. The ratio M = I(V.S)/H(S) evaluates the measure in which the map reflects the distribution of a variable. If M = 1, then I(V.S) = H(S) and the map accurately reflects the spatial distribution of the states of the variable (De Pablo *et al.* **1987).**

Mutual information variation in a hierarchical sectorization

For a given spatial scale, or hierarchical level, the parameter C allows the choice of the best indicators. However this scale need not necessarily be the best for the indicator variable, in spite of C reaching the value of 1. Some ambiguity still remains, as can be seen in Fig. 1 which represents an example of the variation of C in the case of three variables, Va, Vb and Vc, with different frequencies of appearance, in relation to three hierarchical levels in a classification, from which a map is obtained. The variable Va has a high spatial correspondence with the sectors of the first level (R) of the hierarchy and C reaches its maximum value. When the sectors constituting this level are divided into those of the second hierarchical level (T), the variable maintains the same C value. The same occurs with the third hierarchical level (P) considered in the example. In fact, uncertainty concerning the presence or absence of the states of the variable Va in a given sector is the minimum possible in all three cases.

In all three the variable seems equally indicative. However, given its distribution in the map in Fig. 1, it would seem that this variable achieves its 'optimum indicator value' in the first level (R). The same is true for Vb from the second hierarchical level (T) downwards and with Vc from the third (P), supposing that additional lower inferior hierarchical levels have been considered.

In Fig. 1 variation of M values for the same variables are shown. Although the maximum values of this parameter are attained at the same hierarchical levels as those of C, its variation differs from that of C. For each variable the maximum value of M differs relatively little from the other two values, except in the case of Va.

The uncertainty measured by the mutual information, and hence by C and M, appears to be influenced by two factors. First is the frequency of appearance of a given state of a variable in the territory. States with relatively little frequency would be most likely to be completely confined by chance (minimum uncertainty) to a sector. Second is the size of the sector, *i.e.*, the number of observations it contains. In a small sector there is a greater probability that it will be randomly occupied by one state of a variable.

When the sectors of a map respond to a hierarchical structure the position of the sector in that hier-

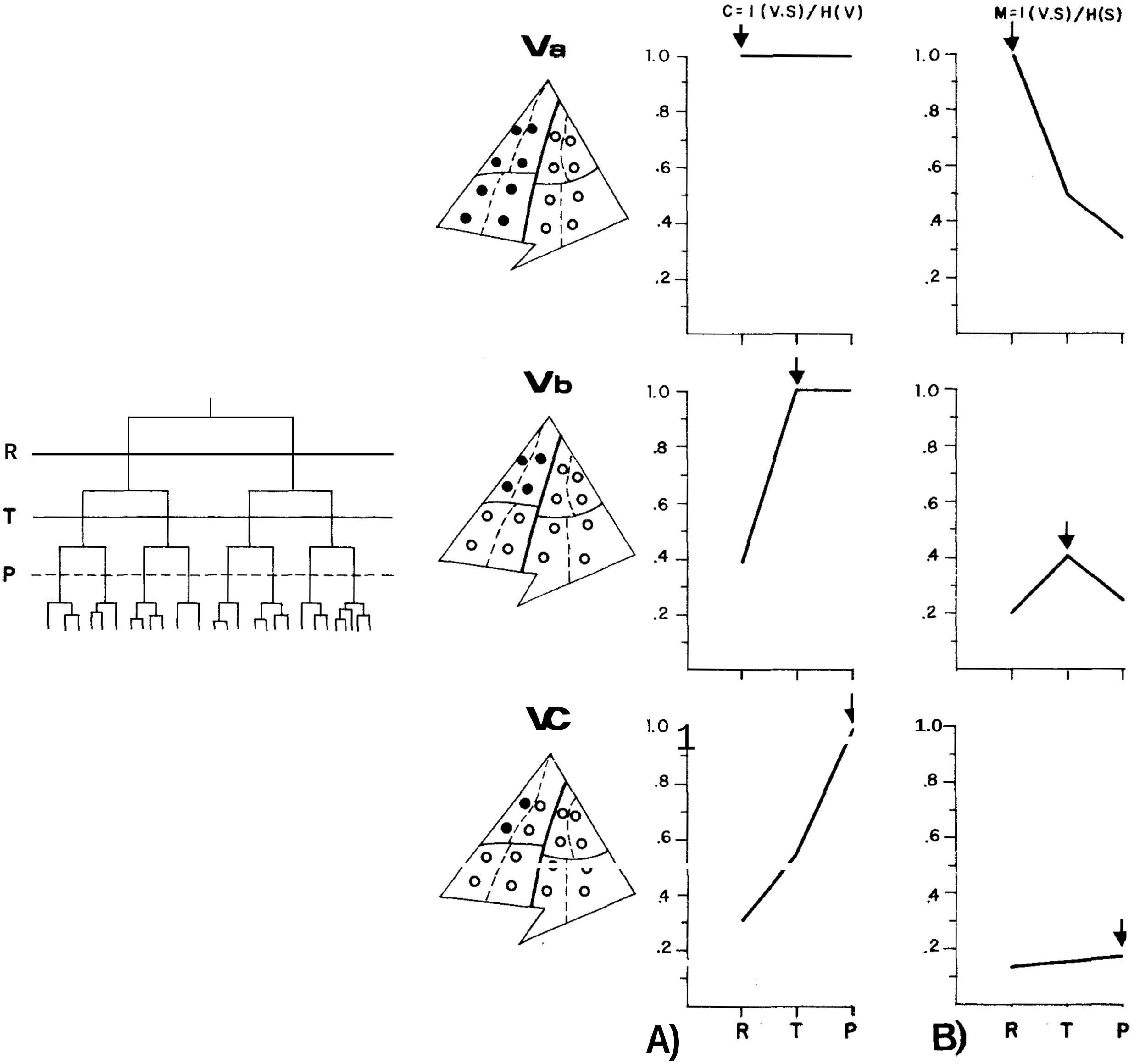

Fig. *1*. Example of variation in *C* = I(V.S)/H(V) and M = I(V.S)/H(S) values. These values are calculated for a set of three variables (Va, Vb and Vc) and three hierarchical levels (R, T, P) of a map. R: two large sectors delimited by a thick line; *T*: four sectors of intermediate size delimited by a thin line and ***P*:** eight small sectors, delimited by a dotted line. Black circles: plots in which variables are present; white circles: plots in which variables are absent.

The *C* variation provides clear informtion concerning the territorial level or scale in which each variable reaches its greatest predictive capacity on a set of sectors (indicated by arrows). Va reaches this capacity for the two sectors of level R; Vb for the four of *T* and Vc for the eight of P.

M also reaches its maximum value in the same sets of sectors as *C* for each variable. Nevertheless, these maxima are not highly differentiated from the other calculated values, except in the case of Va.

archy affects the uncertainty measured by I(V.S), and by *C* and *M*. Each sector of a hierarchical level consists of sectors of a lower hierarchical level. If a variable completely occupies a sector, it also completely occupies its subsectors. Thus, certain variables which appear to be relevant at a lower hierarchical level are really variables 'inherited' from higher levels and exhibit high mutual information

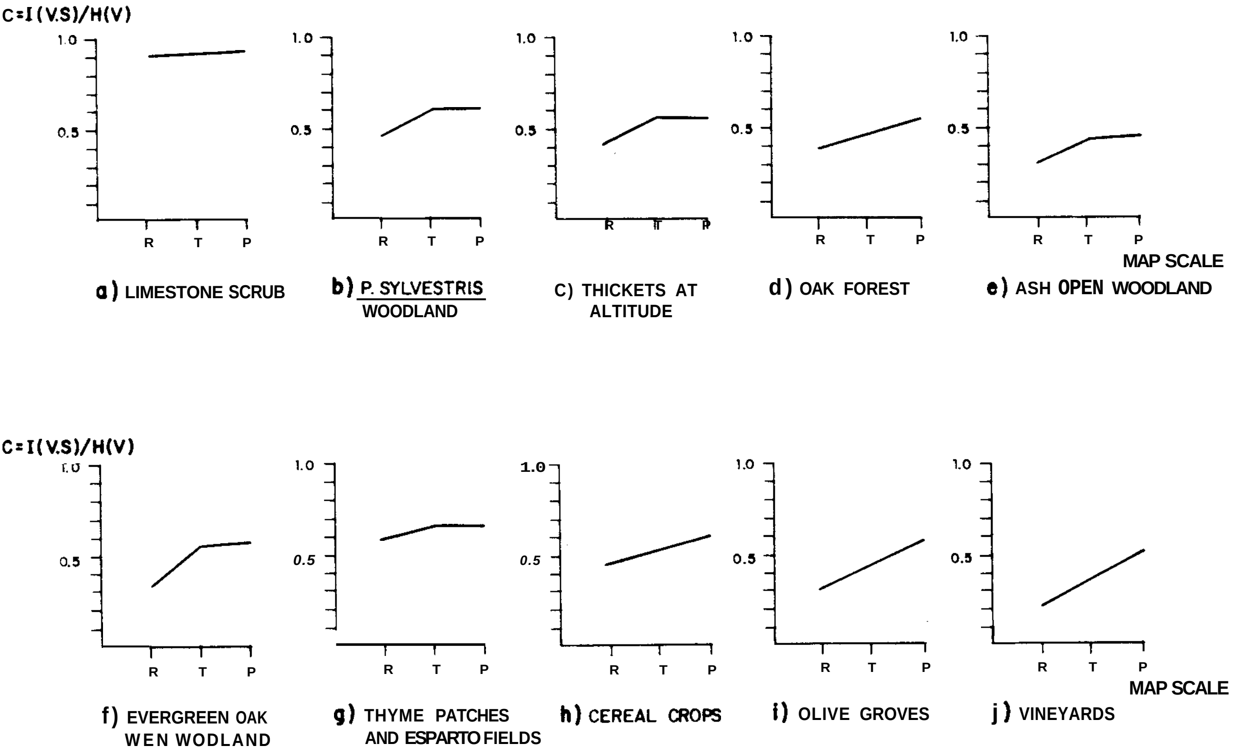

Fig. 2. Examples of *C* spectra of vegetation and land uses variables. The numbers on the abscissae **axis** represent the different levels of cartographic expression studied: five large areas, ***R*****,** which contain the **15** zones of level, *T*, which, in turn, contain the **22** sectors of level ***P*****.**

values with lower level sectors. Nevertheless, the level of highest spatial segregation, with the highest value indicator, may really correspond to higher hierarchical levels.

The analysis of Cvalues for a given variable in relation to the sectors of a different hierarchical level of a classification, provides information about the territorial level or scale at which each variable attains its maximum predictive power for a given set of sectors. The curve resulting from plot *C* versus spatial scale can be named 'spectrum of mutual information values' (in the sense of Margalef, 1968). Observation of this curves permits the identification of indicators of the main territorial units and also those of local situations or lower hierarchical levels.

Cartographic study of Madrid Province

Synthetic cartography

A map of Madrid Province was devised by De Pablo ***et al.*** (1987) based on geomorphological, climatic, and vegetation-land use variables. The map was produced automatically by means of classification analysis and by cartographical projection of classification groups. The map consists of five large areas divided into **15** zones and 22 sectors, representing a cartographic hierarchy with ever increasing detail, *R*, *T* and ***P*****,** respectively.

A total of 675 observations of 1km^2 were plotted and the presence–absence data of 25 vegetation and land-use variables were recorded. These data were used to calculate mutual information spectra.

Indicator values of the variables: mutual information spectra

Figure 2 illustrates examples of *C* spectra of the vegetation and land use variables. Only one variable, limestone scrub (Fig. 2a), has a high indicative value ***(C*** = 0.9). It is also the only one that presents an almost horizontal spectrum. Its descriptive capacity reaches its optimum value in a small detailed map (*R*).

The variables presenting a rectangular spectrum show a spatial variability associated with the zones of intermediate detail on the map (*T*). Of these, *Pinus sylvestris* woodlands (Fig. 2b), high altitude thickets (with *Cytisus purgans,* Fig. 2c), ash and evergreen oak open woodland ('dehesas', Fig. 2e and f), thyme patches and esparto fields (Fig. 2g) have an indicative capacity of $C \geq 0.5$. These variables generally are located in the north of the territory, in the mountain range and its pediment. Other variables with rectangular spectrum (not showed in the figure) such as fruit trees, elm trees, *P. halepensis* and *P. pinea* plantations, juniper groves and urban centers do not have an indicative power at scale T, because $C \leq 0.4$.

Extensive crops (cereal, olive trees and vineyards, Fig. 2h–j) have greater indicative value on detailed maps (*P*), as shown by their diagonal spectra, with $C \geq 0.6$. Boggy meadows, pastures, poplar trees, *P. pinaster* plantations, oak woods, deciduous woods, broom thickets (***Lygos*** *sphaerocarpa),* gypsophile shrubs and acidophils shrubs also possess a diagonal spectra but with lower indicator value (not shown in the figure).

The 25 vegetation and land use variables represent a fairly exhaustive set of the different floral formations and types of land use to be found in the Province of Madrid. Only one appears characteristically on the less detailed scale (*R*). Of the rest, 12 are characteristic of level T and 12 of level *P*. In the map being considered, scales T and ***P*** reveal the spatial variation of the vegetation and land use variables. Both are equally useful in studying such variation.

Discussion

Indicators are generally defined as elements of the territory whose properties (presence, abundance, biological development) allow other properties of an ecosystem which are more difficult to detect to be deduced. Normally the most easily perceived elements of the territory are used as indicators. Thus in many cases the biological components are used (Viktorov *et al.* 1962; Nicolas *et al.* 1979; Bernaldez 1981).

The importance of indicators in ecological cartography lies primarily in the possibility of using them to describe the spatial variability of the relations between variables. They validate map making, by checking its coherence, depending on the cartographic objectives. The nature of the indicators also reveals structural characteristics of territorial organization (the most important factors which govern their spatial differentiation and the nature of this differentiation). In cartography, the relation between indicator and indicated object is complex and the search for indicators resembles the problem of identifying and outlining the ecological structure of the territory. Indicators can therefore be taken as examples of logical steps from the 'phenosystem' to the 'cryptosystem' (Bernaldez 1981), which show the relations between landscape elements and the types of interacting systems underlying the landscape. This same idea of structural and functional process indicators can be found in Viktorov *et al.* (1962), Naveh and Lieberman (1984), Westman (1984) and in general, in the field of 'landscape ecology' and 'ecology applied to environmental planning and territorial management'.

The choice of an accurate indicator is conditioned by two factors: (i) the number and nature of states of the indicator variable and (ii) the number and spatial disposition of maps units. Usually, however, the number and nature of variable states and map sectors are chosen 'a priori', *i.e.*, without taking into account its 'a posteriori' correspondence, on which the indicative capacity is based. **A** variable with two states, for example, may be a bad indicator and with three states a good one. In addition, given a number of states, a variable would be a good indicator for a map with three sectors and a bad one for the same map with five sectors. If these possibilities are not tested the search for indicators remains incomplete. The use of *C* and *M* parameters in a spectral form could override this difficulty.

This paper shows an example of how this optimization could be undertaken. The C value for a given variable is observed at three differents spatial scales

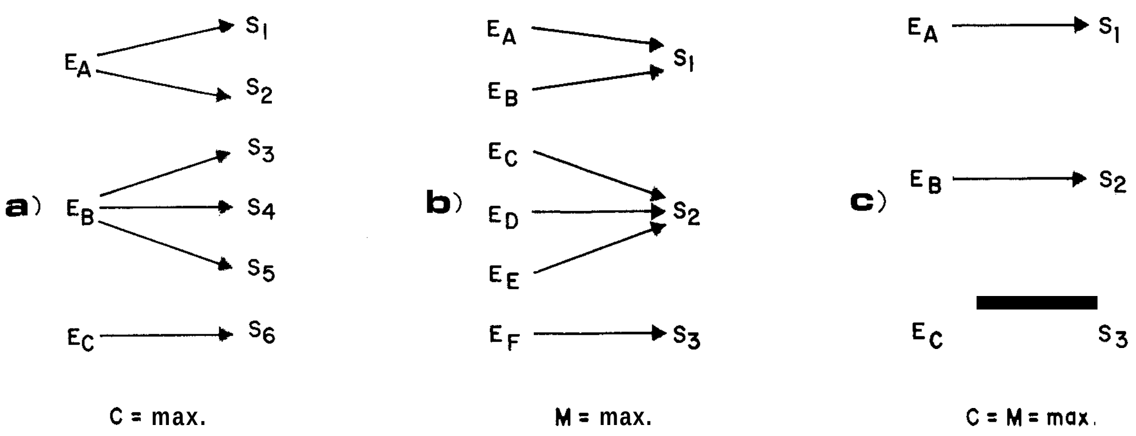

Fig. 3 Diagram representing three possible relations between an indicator variable (E) and an indicated object, in this case map sectors (S). The arrows indicate the correspondence between the indicator states (letters) and classes of the indicated object (numbers). In (a) the indicator is as close as possible to maximum *C* value. However its indication is ambiguous, as only one sector is clearly indicated. In (b) the map isolates the variable states as well as possible, maximum value of *M*. The indication is again ambiguous, as various states of the indicator indicate the same sector. In (c) the 'recognition' of indicator and indicated object is mutual, $C = M$ = maximum possible value, and indication is unambiguous. The aim is to obtain type (c) indicators with the mutual information values spectrum – either of *C* or *M*.

(in the way of diversity spectra proposed by Margalef, 1968). This allows the choice of the scale on which the variable, with these states, is more predictable in the spatial distribution of the maps units.

The optimization obtained through mutual information spectra does not only affect the level of ecological-geographical expression of a variable which assumes different states. For a given level of expression or spatial scale, it is also possible to optimize the set of the states of the variable which reaches the highest predictive value. A procedure may therefore be designed to redefine both the variable's states and map sectors employed, in such a way that it is possible to precisely determine the indicative possibilities of each. In this paper only the descriptive possibilities of a single set of variable's states (presence or absence) have been tested in relation to different sectorizations of a map. Spectra of M are not used in this paper, due to the impossibility of considering other sets of states. This could explain the low indicative value of some of the given variables. The assessment of their indicative capacity and of their role in the ecological differentiation of the territory remains, therefore, in some ways incomplete.

Figure 3 reveals that parameter C is less ambiguous than M in the detection of reliable indicators at a given level of perception. They only coincide in the case of an ideal indicator which has both simultaneously high certainty and importance (Viktorov et *al.* 1962; Bernaldez 1981). For this reason a 'one to one' correspondence between all the indicator states and the indicated object classes may exist. Both should have the same number of states and classes. It is sufficient for the indicator to have one of its states clearly corresponding to a class of the indicated object for that state to be considered an indicator (Fig. 3a). The indicated object, on the other hand, appears to be more restrictive in recognizing an indicator as such. It must be globally recognized by the indicator, *i.e.*, none of the indicator states point to more than one of the indicated object classes (Fig. 3b). Finally in the case of the ideal indicator the identification between indicator and indicated is reciprocal (Fig. 3c).

Conclusion

Information theory parameters are particularly useful in the study of the relation between indicator and indicated object. In the context of ecological cartography, the indicator is represented by the different territorial variables considered, while the indicated object would be the network of relations between the territory's components. The relation between both is similar to that which is established, in an information channel, between the input symbols – states of a variable – and output symbols – sectors of a map which synthesize the network of the mentioned relations – and consists of the probability with which a given input symbol produces a given output symbol. The set of these conditioned probabilities, for all the input and output symbols can be considered to constitute the information channel (Shannon and Weaver 1949; Abramson 1963).

The mutual information represents a measure of the quantity of information transmitted without noise across this channel (Phipps 1981). The noise, quantified by the values $H(V|S)$ or $H(S|V)$, depends on the structure of the studied territory, *i.e.*, the correspondence between sectors and variables. The present study proposes modifying the values $H(V)$ and $H(S)$ by means of redefining variable states and map sectors respectively, in order to minimize values $H(V|S)$ and $H(S|V)$, and maximize $I(V.S)$. This is equivalent to reducing noise to a minimum, in which case the indicator provides the maximum information about the indicated object.

The method proposed here allows the assessment of the degree (from 0 to 1) to which a variable, with a given set of states, is a good indicator at a given spatial scale. This allows the choice, by means of C spectra, of the best set of indicators for this spatial scale. The assessment, however, seems incomplete, given that the nature and number of states of the variable are of great importance to determine its predictive value. Thus, this assessment must be carried out on the variable states and map sectors at different spatial scales simultaneously, employing C and M joint variation at the same time. This joint variation allows the most analogous indicators to those shown in Fig. 3c to be chosen. This is possible from a methodological point of view, but further investigation is needed for practical purposes.

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Appendix

Procedure followed for calculating the different parameter values of the total entropy theorem.

u_{ij} (i = **1, 2, . . .**, r; j = **1, 2, . . .**, c) is the number of observations in which the state i of a variable appears in the sector j of a map, the matrix representing the distribution of the variable states in a set of sectors would take the form:

$$\begin{array}{lllll}
u_{11} & u_{12} & \ldots & u_{1c} & u_{1\cdot} \\
u_{21} & u_{22} & \ldots & u_{2c} & u_{2\cdot} \\
\cdot & & & & \\
u_{r1} & u_{r2} & \ldots & u_{rc} & u_{r\cdot} \\
u_{.1} & u_{2.} & \ldots & u_{.c} &
\end{array}$$

where c is the total number of sectors and *r* the total number of variable states. In this study, r = **2** in all cases (presence or absence). The probability of finding state ***i*** in sector *j* is given by p_{ij}:

$$P_{ij} = u_{ij} / \sum_i \sum_j u_{ij}\,;\ p_{i.} = u_{i.} / \sum_i \sum_j u_{ij}\,;\ p_{.j} = u_{.j} / \sum_i \sum_j u_{ij}$$

where u_{ij} represents the total number of observations, $p_{i.}$ represents the probability of finding state ***i*** of the variable in all the territory **and** $p_{.j}$ the probability of finding sector *j* in all the territory. The following expressions are taken from the entropy employed by Shannon (Shannon and Weaver **1949):**

$$H(V) = -\sum_{1} p_{i.} \log_2 p_{i.}$$

$$H(S) = -\sum_{j} p_{.j} \log_2 p_{.j}$$

$$H(V.S) = -\sum_{i}\sum_{j} p_{ij} \log_2 p_{ij}$$

$$H(V|S) = \sum_{j} [-\sum_{i} p_{ij} \log_2(p_{ij}/p_j)]$$

$$H(S|V) = \sum_{i} [-\sum_{j} p_{ij} \log_2(p_{ij}/p_i)].$$