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Identifying structural self-similarity in mountainous landscapes

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Abstract

Digital elevation model data were used to partition a mountainous landscape (northwestern Montana, **USA)** into watershed/hillslope terrain units at several different spatial scales. Fractal analysis of the perimeter to area relationships of the resulting partition polygons identified statistical self-similarity across a range of spatial scales (approximately four orders of magnitude in partition area). The fractal dimension was higher for a relatively complex fluvially-dominated terrain than for a structurally simpler glacially-dominated terrain (1.23 vs. 1.02, respectively). The structural self-similarity exhibited by this landscape has direct implications in scaling up ecosystem process models for landscape to regional simulations.

Introduction

In hilly or mountainous landscapes, the strong influence of topographically controlled microclimate on surface hydrologic and ecologic processes often results in a geographic pattern that closely follows the pattern of the terrain. The terrain, itself, often exhibits a spatially recursive pattern; at each increase in resolution, a greater extent of the stream network is represented and the landscape is partitioned into smaller and smaller watersheds. Watersheds are in turn composed of a pair of opposing hillslopes, which represent the fundamental landscape entity in this schema. **A** change in the scale of observation does not change the basic morphological characteristics of the watersheds or hillslopes, thus making this recursive watershed pattern scale-invariant. Fractal geometry provides a convenient method of describing the characteristics of objects that are scale-invariant, or self-similar (Mandelbrot, 1982).

In applying this concept of self-similarity to landscapes, two research questions are raised. First, over what range of spatial scales does the landscape of interest display structural self-similarity in terms of watershed morphological characteristics? Second, does the landscape also display functional self-similarity in terms of ecological processes over this same range of spatial scales (*i.e.*, does a small watershed function similarly to a large watershed)?

The objective of this paper is to examine the first question on structural self-similarity for a large river basin (the Seeley-Swan Basin, 1200 km^2) in northwestern Montana. Several watersheds were analyzed with different resolutions of partitioning to determine:

1) the range of scales displaying self-similarity; and

2) any differences in fractal dimension under different geomorphological regimes.

This work is part of an ongoing project on the ecological process modeling of forest evapotranspiration and primary productivity by investigators at NASA-Ames Research Center and the Universities of Montana and Toronto (Band et al., 1991; Running et al., 1989).

Fractal properties of landscapes

The term fractal was coined by Mandelbrot (1977) to describe any function for which the ''Hausdorff-Besicovich'' dimension exceeds the topological dimension (e.g., 0 for points; 1 for lines; 2 for areas). The fractal dimension, D, is a single, non-integer parameter that gives a useful measure of the complexity or roughness of the spatial pattern. Linear fractal functions vary between 1.0 (for a straight line) and 2.0 (a line so irregular and rough that it effectively fills the whole of a two-dimensional space) (Mandelbrot, 1982). Surfaces vary between 2.0 (infinitely smooth) and 3.0 (infinitely crumpled).

Ideal fractal objects reveal exact copies of themselves with each scale change. Most natural objects are not strictly self-similar but rather statistically self-similar. The objects "look" similar because the statistics of their surface are constant across a range of spatial scales. Many landscape features and other environmental data have been shown to exhibit fractal behavior (constant D) across a wide range of scales (Goodchild and Mark, 1987; Burrough, 1981). Other environmental systems show a self-similarity over a few widely separated scales connected by transition zones (Krummel *et al.,* 1987; Mark and Aronson, 1984; Burrough, 1981).

The range in scales is determined by the controlling geophysical processes that have or continue to shape the landscape. In the case of a fluvially dominated landscape, the coarsest scale may be delimited by regional geology, while the finest is determined by the diffusive process of water itself. Determinations of the range in spatial scales over which systems display self-similarity provides a means of identifying scales at which fine-scale processes create larger scale patterns (Milne, 1988).

Methods

Three small watersheds were selected for study: the North and South Forks of Elk Creek (17 km² and 39 km² in size, respectively) and Soup Creek (15 km², a tributary of the Swan River). The larger Swan River basin (1000 km²) was also analyzed for comparison. A digital elevation model representing the landscape was partitioned using software devised by Band (1989, 1986), which recursively partitions the drainage network into hillslope units. 30 meter USGS digital elevation model (DEM) data for the three small watersheds and **3** arc-second DMA digital terrain model (DTM) data (interpolated to a 100 m grid) for the larger Swan River basin were used.

The three smaller watersheds were partitioned into hillslopes of varying resolution. For instance, the North Fork of Elk Creek (NFEC) was partitioned into 6, 10, 14, 18, 30 and 66 hillslope units. Not every hillslope unit is further divided with each partitioning step. For example, when 6 hillslopes were repartitioned into 10 hillslopes, 4 of the hillslopes remained the same, while 2 of the original were repartitioned into 6 new hillslopes. Increasing the number of partitions, decreases the mean size of the hillslopes (*e.g.*, the mean area of the original 6 partitions is 2.8 km², compared with 0.25 km² for the 66 partition scheme).

Natural planar shapes (*i. e.*, mapped polygons or landscape patches) can exhibit fractal behavior as demonstrated in the relationship between shape area and the perimeter (Mandelbrot, 1977; Lovejoy, 1982). The perimeter of a patch is related to the area A of the same patch by $P \cong \sqrt{A^D}$ (transforming gives $2/D \log P = \log A$). For simple Euclidean shapes (*e.g.*, a square) $P \cong \sqrt{A}$ and $D = 1.0$, while more complex shapes with convoluted perimeters, $P \cong A$ (*i.e.*, shapes that are space-filling) as D approaches 2.0. The fractal dimension, D, of the partitions was approximated by the slope of the log-log plot of partition area vs. perimeter (*i.e.*, slope $= 2/D$, thus $D = 2/\text{slope}$). As the partition data was in a raster format, the perimeter/4 was used as way to compensate for the square grid cells (*i.e.*, $P \cong 4\sqrt{A^D}$ or $P/4 \cong \sqrt{A^D}$). Area-perimeter relationships have been used to explore the fractal geometry

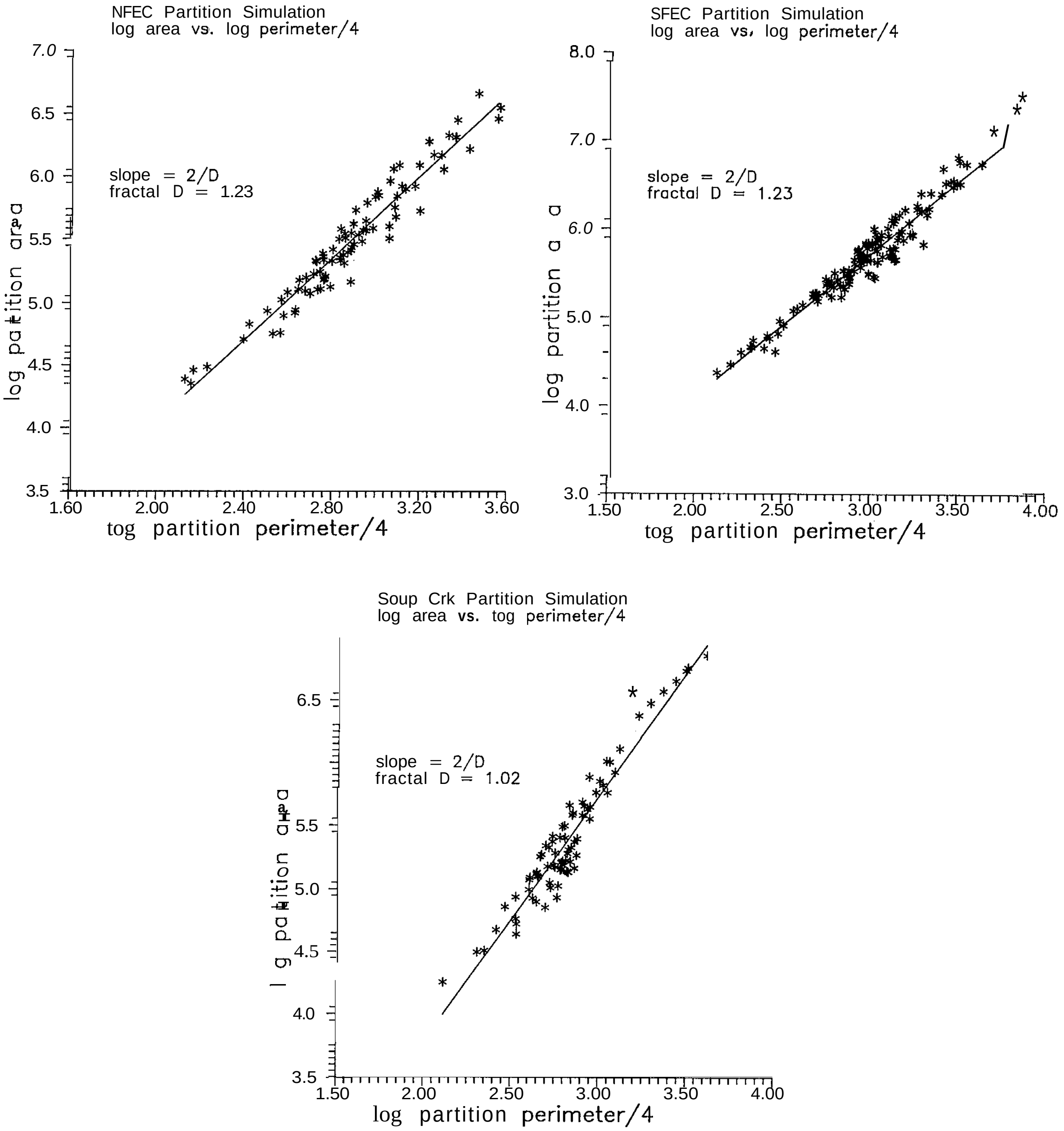

Fig. 1. **Plot of log area vs. log perimeter for the three small watershed study areas. The fractal dimension, D = 2/m. Area is in m^2, perimeter in m.**

Fig. ZA. **North Fork Elk Creek study area (6, 10, 14, 18, 30, 66 partitions).**

Fig. **IB. South Fork Elk Creek study area** *(2,* **6,** 14, 38, **50,** 82, **102 partitions).**

Fig. 1C. **Soup Creek study area** *(2,* 10, *22,* 52, 72 **partitions).**

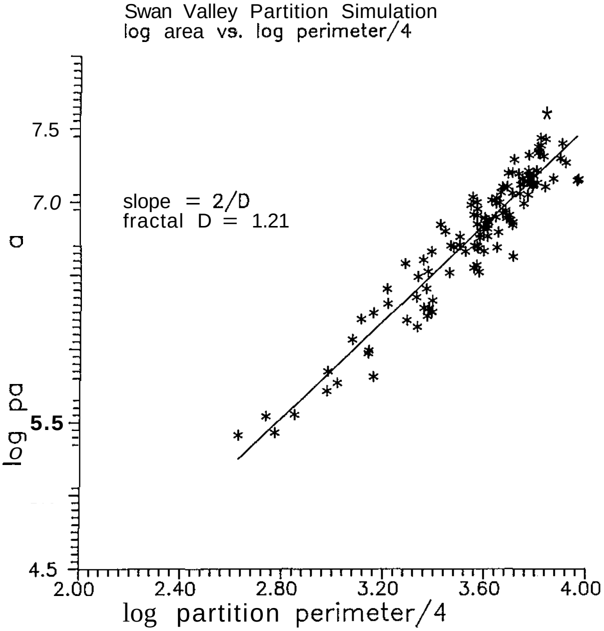

Fig. 2. Plot of log area vs. log perimeter for the Swan River basin study area (180 partitions).

of clouds (Lovejoy, 1982) and digital land-use data (Milne, 1988; Gardner *et al.*, 1987; Krummel *et al.*, 1987).

Results

The analysis undertaken for the three small watershed study areas shows that the perimeter-to-area relationship for the partitioning is strongly linear with R^2 values > 0.90 (Figs. 1A–1C). Large hillslopes (*i.e.*, NFEC broken into only ***6*** partitions, mean area = 2.8 km^2) had a similar value for D as small hillslopes (*i.e.*, NFEC broken into 66 partitions, mean area = 0.25 km^2). The Swan River basin was analyzed for only one partition simulation (180 partitions, the valley bottom was excluded from the analysis) but the resulting partitions varied greatly in size (Fig. 2). The four study areas show self-similarity across the entire range of spatial scales examined (approximately three orders of magnitude in perimeter length from 0.4 km to 40 km and four orders of magnitude in partition area from 0.03 km^2 to 27 km^2).

The fluvial landscape characteristic of the North and South Forks of Elk Creek (NFEC and SFEC) had a fractal dimension of D = 1.23. While the alpine glacial topography of Soup Creek, characterized by more elongated tributary basins and rounded glacial cirques, had a significantly lower fractal dimension of D = 1.02 ($p < 0.001$, two-tailed test). From this initial analysis it appears that the fractal geometry of the hillslope partitioning is different for the two geomorphological regimes.

However, the entire Swan River basin (which Soup Creek is a tributary **of)** had a fractal dimension of 1.21, similar (though still significantly different, $p < 0.05$) to that found for the NFEC and SFEC study areas but significantly different from the Soup Creek result (Fig. 2) ($p < 0.001$). The Swan River basin is bounded by the highly glaciated Swan and Mission Ranges and it was hypothesized that its fractal geometry would behave similarly to that of Soup Creek. The difference in sampling resolution between the USGS DEM **(30** m grid cell; used for the smaller watersheds) and the DMA DTM (100 m grid cell; used for the larger river basin) may be responsible for the difference in D and needs further investigation.

Discussion

Using the perimeter to area relationship of hillslope units as a measure of landscape structure, the three watersheds display structural self-similarity across a range of partitioning schemes (*i.e.*, a small first-order hillslope "looks" like a larger higher-order hillslope). No distinct endpoints or transition zones were observed, rather the fractal dimension remained constant across the range of scales analyzed. Mark and Aronson (1984), in an analysis of digital elevation models of several areas in the U.S., found that over short scales (below 0.6 km) many of the geomorphic surfaces analyzed resembled fractal surfaces. Conversely, a topographical surface for a mountainous district in Colorado, like the mountainous landscapes of Montana analyzed in our study, was characterized by a consistent dimension (D = 2.28) across a broad range of scales (0.4–11 km, the entire range of scales examined).

The values of D for the NFEC and SFEC watersheds and the larger Swan basin compare favorably with the dimensions (*e.g.*, D = 1.2–1.3) found for river and coastline lengths (Mandelbrot, 1982). The fractal dimension for Soup Creek is significantly lower (D = 1.02), presumably a reflection of a difference in the underlying geomorphological regime (*i.e.*, alpine glacial vs. fluvial). **A** greater number of watershed basins need to be examined to further elucidate the relationship of D to different geomorphological regimes.

Implications to scaling ecosystem models

The structural self-similarity exhibited by the topography may have direct implications in landscape ecological modeling. One of the major challenges that must be addressed is the proper scaling of point or stand process models for ecosystem process simulation at landscape to regional scales (Turner *et* al., 1989). One stumbling block is characterizing the spatial heterogeneity symptomatic of larger spatial scales. **As** the terrain is more finely divided, smaller hillslopes can be delineated that are generally more uniform in such surface parameters as vegetation, soils, and microclimate. Correspondingly, the larger hillslopes, composed of a number of smaller hillslopes, will generally display a greater heterogeneity of surface parameters. How does one sample and represent the full distribution of surface parameters of a landscape or region?

One approach is to partition the landscape into distinct land elements *(i.e.*, watershed or hillslope units), which represent relatively homogeneous but still ecologically meaningful areas (Band *et al.*, 1991). This stratification attempts to capture as much of the full scene information as between-units variance, with a consequent reduction in within-unit variance (Band, 1986; 1989). These partitions provide the template to organize the various model parameters allowing for spatial aggregation and consequent data reduction; decreasing the number of partitions further increases simulation model efficiency.

This landscape partitioning scheme has been shown to display statistical self-similarity across a range of spatial scales. If the underlying process determining the landform pattern also controls the ecosystem functioning, then self-similarity in ecosystem functional attributes can also be expected. It is critical that the self-similarity in ecosystem functional properties must be directly established and not simply assumed from the self-similarity in landscape form.

The existence of self-similarity in ecological properties for the study area in northwestern Montana has not been conclusively established to date and is the focus of continuing work. If the landscape partitions also display functional self-similarity of selected ecosystem processes over the same range of scales, then the models parameterized for sub-watersheds (*e.g.*, experimental, highly-instrumented basins) can be extrapolated to larger scales for regional estimation (*e.g.*, net primary production or evapotranspiration for an entire river basin). The landscape can be generalized by limiting partitioning and simulating to the major watershed/hillslope units (*i.e.*, 2 rather than 66 partitions) as a means to scale up for regional ecosystem process modeling.

Within the context of this landscape partitioning scheme, fractal analysis provides two pieces of information of potential importance to scaling ecosystem models. First, the fractal dimension provides a potential link connecting landscape structural characteristics and ecosystem functional attributes. The fractal dimension could be used as one criterion to stratify regions into different landscapes (*i.e.*, areas that are controlled by different geomorphic regimes) that require separate ecosystem model parameterization. Second, assuming that there is a relationship between landscape structure and ecosystem functional attributes, fractal analysis identifies the ranges in scale of statistical self-similarity in landform structure and thereby provides a method for determining the endpoints for ecosystem model extrapolation.

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