Landscape Ecology vol. 9 no. 4 pp 279-286 (1994)
SPB Academic Publishing bv, The Hague

Fractal dimension estimates of a fragmented landscape: sources of variability

Alain Leduc[1], Yves T. Prairie[2] and Yves Bergeron[1]
[1]Groupe de Recherche en Ecologie Forestière (GREF); [2]Groupe de Recherche Interuniversitaire en Limnologie (GRIL), Ddpartement des Sciences Biologiques, Université du Qukbec a Montrkal, C.P. 8888, Succursale 'A', Montrkal, Qukbec, Canada H3C 3P8

Keywords: Semivariogram, anisotropy, grain size effect, scale effect

Abstract

Although often seen as a scale-independent measure, we show that the fractal dimension of the forest cover of the Cazaville Region changes with spatial scale. Sources of variability in the estimation of fractal dimensions are multiple. First, the measured phenomenon does not always show the properties of a pure fractal for all scales, but rather exhibits local self-similarity within certain scale ranges. Moreover, some sampling components such as area of sampling unit, the use of a transect in the estimation of the variability of a plane, the location, and the orientation of a transect all affect, to different degrees, the estimation of the fractal dimension. This paper assesses the relative importance of these components in the estimation of the fractal dimension of the spatial distribution of woodlots in a fragmented landscape. Results show that different sources of variability should be considered when comparing fractal dimensions from different studies or regions.

1. Introduction

Since its popular introduction by Mandelbrot (1982), fractal geometry has been increasingly used in several fields of application. In ecology, fractal theory has been used mostly as an analytical tool either to identify characteristic scales (Frontier 1987), structural hierarchy (Burlando 1990; Collins and Glenn 1990) or to assess chaotic behavior (Sugihara and May 1990) of phenomenon showing a spatial distribution or a temporal dynamics. One of the main appeals of fractals to ecologists lies in their ability to summarize the complexity and heterogeneity of a spatial or temporal distribution in a single value, the fractal dimension (FD), that is purportedly independent of scale.

Although most studies have focused on reporting fractal dimensions for various environmental variables (Burrough 1983a, b; Palmer 1988; Milne 1990), some recent work has begun to provide comparisons of fractal dimension estimates of a particular variable in several locations (*e.g.*, Logan and Wilkinson 1990) or among different portions of a single system (Krummel *et al.* 1987). These comparative studies highlight the problem of determining the uncertainty associated with fractal dimension estimates. Without knowledge of this uncertainty, the comparison of fractal dimensions among studies is difficult, at least from a statistical point of view. This warning is amply justified given the paucity of studies documenting the influence of the various components of the sampling involve in the estimation of fractal dimensions. The size of the sampling unit (*i.e.*, the grain size), the relative positioning of the sampling grid or transect, and the orientation of the transect are all potential sources

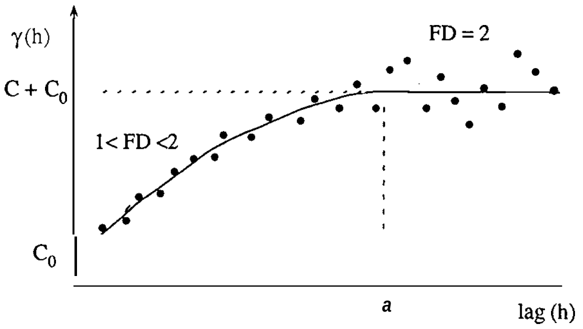

Fig. 1. Components of a theoretical semivariogram. (C,) the nugget variance, (a) the range, the sill (C + C,) and fractal dimension (FD) associated with different phases in the increase of the semi-variance. The lag (h) refers to the various inter-point distances at which we calculated the semi-variance γ(h).

of variability in the estimation of fractal dimension (Burrough 1981).

Our study attempts to quantify the importance of several sources of variability in the estimation of the fractal dimension of the forest cover of a rural landscape. We examined four sources of variability: 1) the lack of self-similarity (*i.e.*, the effect of scale on FD), 2) the use of transects in determining the fractal dimension of a surface, 3) anisotropy (*i.e.*, the occurrence of major axis of variation in space), and 4) the grain size effect (*i.e.*, the effect of the size of the sampling unit).

2. Methods

2.1 Semivariogram and fractal dimension

We estimated the fractal dimension using the relationship between the fractal dimension of a series and the slope of the corresponding log-log semivariogram (Burrough 1983a; Palmer 1988). We chose this method over the more familiar box-counting method (see Milne 1990) because it is better suited to test the influence of anisotropy and to quantify the lack of self-similarity. **A** semivariogram displays the variability (γ(h)) among measurements of a variable separated by distance (or time unit) as a function of that instance (h). Here, the measure of variability is calculated for observations separated by a distance h as:

$$\gamma(h) = \sum_{i=1}^{N(h)} [z(i) - z(i+h)]^2/[2\ N(h)]$$

where z(i) is the value of the variable z at point i, z(i + h) is its value at a point distant by h, and N(h) is the number of observation points separated by a distance h. Generally, γ(h) increases gradually with distance separating the observations up to a maximum value (the sill) representing the maximum spatial variance of the system (Fig. 1). The distance at which the sill is reached represents the range of variation, *i.e.*, the distance within which observations are spatially dependent (Trangmar *et al.* 1985; Burrough 1987). Finally, although the intercept of a semivariogram (*i.e.*, the variability between measurements separated by a distance h—0) should be zero, it rarely reaches zero in practice. This 'nugget variance' is related to instrumental errors and sampling scale (Burgess and Webster 1981; Trangmar *et al.* 1985; Robertson 1987). On a double logarithmic scale the slope (m) of a semivariogram is related to the fractal dimension (FD) as FD = (4 – m)/2 (Burrough 1983a). **A** graphical representation of the variations of FD values as a function of scale (inter-point distances) has been called a fractogram (Palmer 1988).

2.2 Data source

The data used to examine the influence of various sampling components to estimate FD represents the variation in forest cover (here defined as the fraction of the surface area covered with forest) of an agricultural landscape located in the southernmost region of Quebec, the Cazaville county. The analyses were conducted from a digitized map (1:20,000) with a grid composed of 30,000 pixels of one hectare each. Each pixel was given a value according to whether the included area was predominantly forested (1) or not (0).

The variation in forest cover illustrated in Fig. 2 was determined by calculating the proportion of forest cover (proportion of pixels coded 1) with a local operator (*i.e.*, a window) of 9 ha (3 x **3** pixels) moving along each pixel of the map. The flexibility of a mapped variable (*e.g.*, percentage

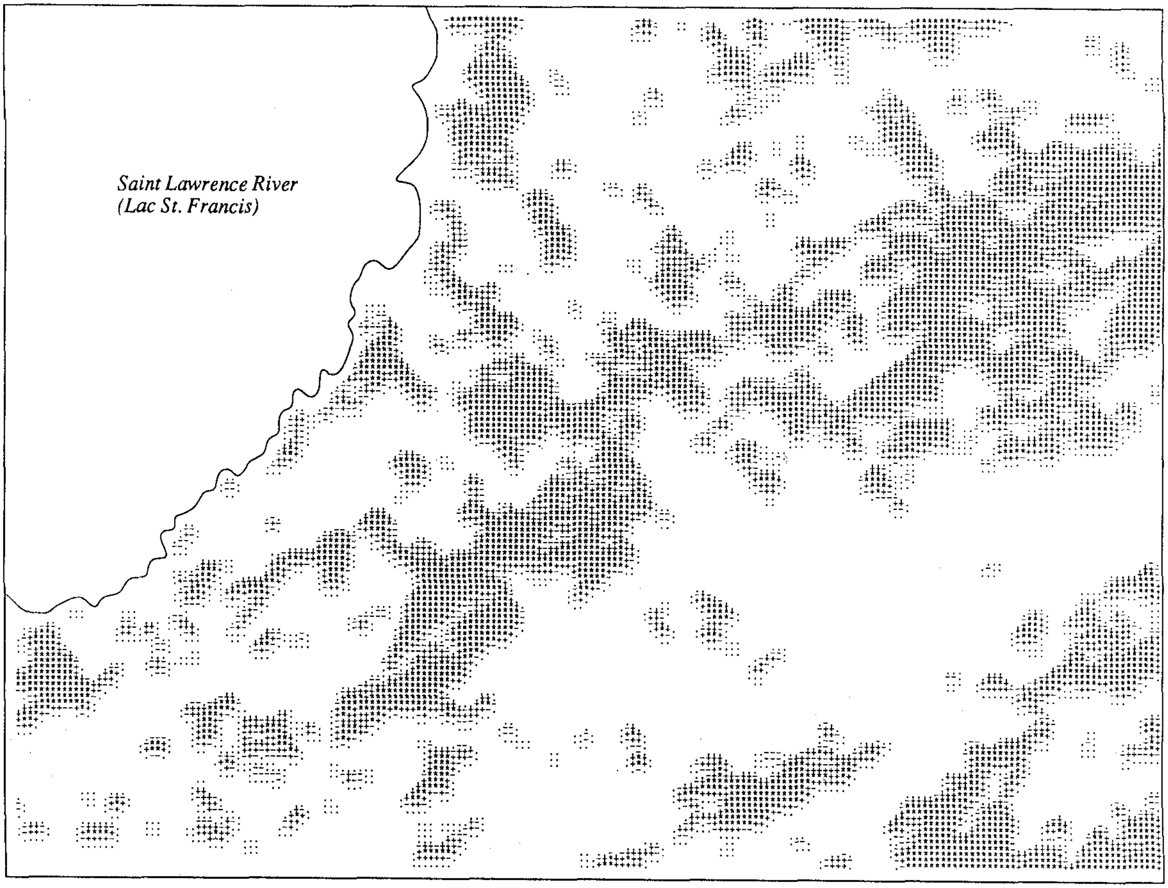

Fig. 2. Map of forest cover of Cazaville County. Forest cover was calculated for 9 ha area (3 X 3 pixels) and expressed in percent. Cartographic units were coded as follows: 20 to 40% (.), 40 to 60% (–), 60 to 80% (+), or more than 80% (*) of the forested area.

forest cover), in contrast to variables derived from sampling programs (*e.g.*, chemical properties of soil), allows easier testing of the effect of various sampling components on a well-known population of map units covering an entire study site.

2.3 Data analysis

To assess the fraction of the variability in FD estimates on a surface due to the use of a one-dimensional transect, we established a distribution of FD values from the analysis of 49 transects oriented east-west covering the map systematically. **A** semivariogram was then first constructed for each transect which was formed by a stripe of 200 contiguous points sampled on the map. **As** semivariograms were often nonlinear, we fitted second-degree polynomial regressions by ordinary least-squares in order to derive the slope and hence the FD value. The slope at any given lag of h is then the first derivative of the polynomial regression equation. When the quadratic term was not significant ($p > 0.05$), we used simple linear regression analysis. For each inter-point distance (lag(h)), we have a set of 49 FD values which allowed us to calculate a confidence interval.

Another potential source of variability in FD estimates results from the presence of spatial patterns with a particular orientation (*i.e.*, anisotropy; Burrough 1981). We examined the importance of anisotropy in our landscape by establishing different semivariograms and corresponding fractograms according to four major axes (E-W, SW-NE, N-S, and SE-NW).

Lastly, we examined the influence of the size of

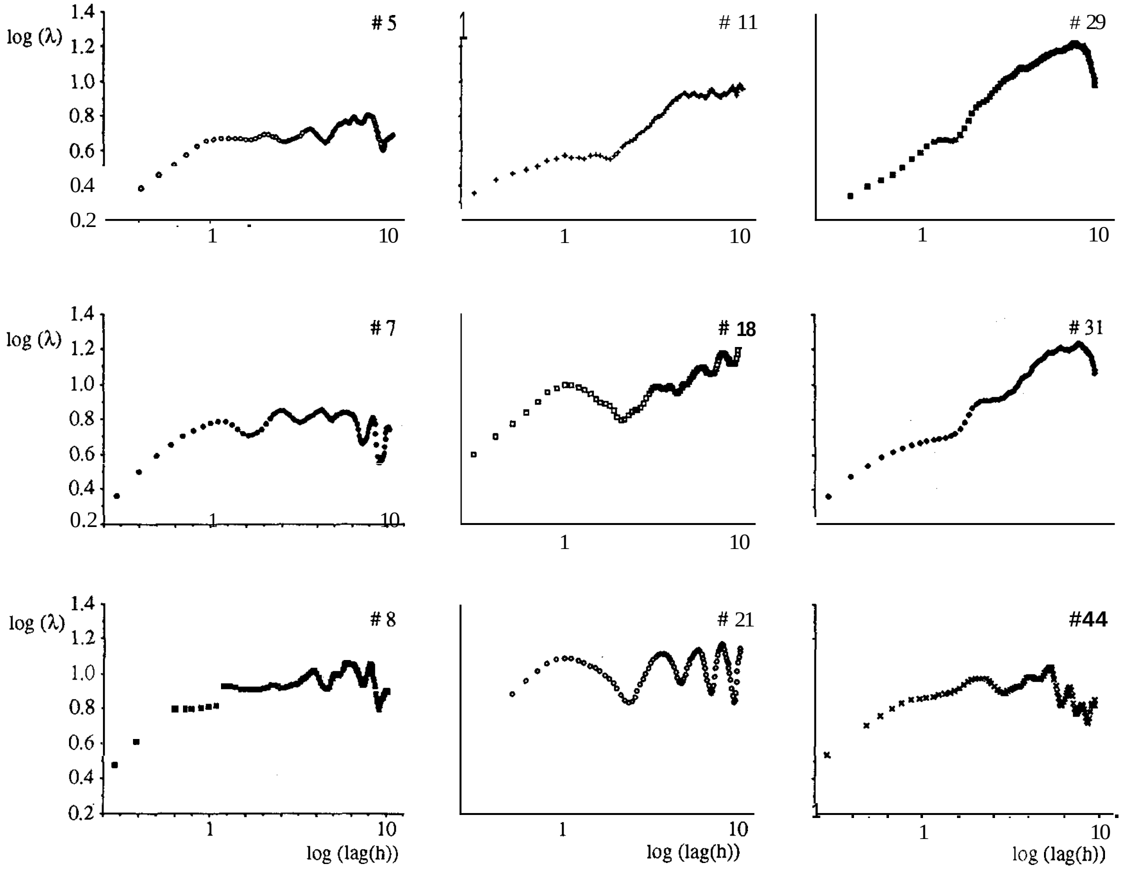

Fig. 3. Semivariograms of forest cover variations observed along East-West transects distributed on the map of Cazavill County.

the sampling unit (*i.e.*, grain size effect) by comparing the semivariograms and fractograms derived using the 9-ha local operator to semivariograms and fractogram derived with a 25-ha operator (5 x 5 pixels). All semivariograms were obtained using the program provided by Journel *et* ***al.*** (1985). All regression analyses were calculated with STATVIEW (Feldman *et al.* 1987).

3. Results and discussion

Several semivariograms derived from the analysis of the 49 east-west transects are illustrated in Fig. 3. Semivariances were calculated only for distances smaller than 15 km as there were too few observations for distances approaching the maximum length of the transects (Journel and Huijbregts 1978). Of the 49 transects, only **a** few (*e.g.*, nos. 29 and 31) exhibited constant slope across all scales. This implies that the fractal dimension of the forest cover is generally not independent of spatial scale. It also implies that changes in forest cover within a landscape are not self-similar as in a pure fractal. Burrough (1981, 1983a) and Palmer (1988) have obtained similar results in their studies of soil characteristics and vegetation, respectively. They concluded that natural phenomena rarely present absolute self-similarity across large scales but rather exhibit local self-similarity within certain scale ranges. This has led them to study variations in fractal dimension as a function of scale. **A** simple visual inspection of our semivariograms clearly shows that, although the semivariance generally increases with scale (distance), many of them exhibit a sudden drop at a distance of 20 sampling units ($10^{1.3}$ or 2 km). This scale represents the average linear size of the patches (of forest and non-forest

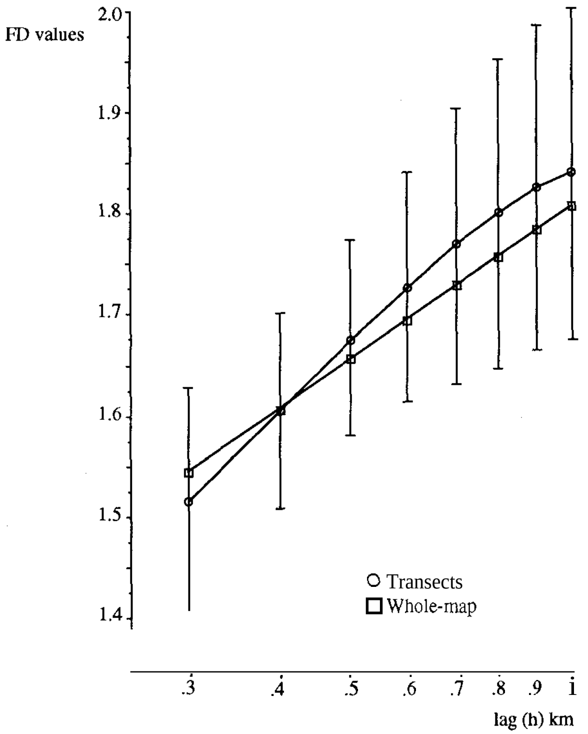

Fig. 4. Fractograms of forest cover variations in Cazaville County at scales ranging from **0.3** to 1 km. Circle points show the mean values **of** the 49 transects and T-bars denote 95% confidence limits. Square points represent the FD values obtained from an analysis of the entire map in the East-West axis.

areas) of our landscape when it was scanned in the East-West axis.

An upper bound of l-km was retained to illustrate the distribution of the fractal dimension estimates (Fig. 4) because most of our semivariograms appears to reach their first sill at scales near l-km. Even within this narrow range of spatial scale (< l-km) however, most semivariograms exhibited significant curvilinearity and hence produced fractograms showing a continuous increase of FD values with scale (Fig. 4). This indicates that the fractal dimension of the forest cover remains influences by scale. Values vary from 1.51 for interpoint distances of 0.3-km to 1.84 for l-km distances. Unsurprisingly, observations are more closely dependent at smaller scales. An ANOVA on all observations (49 transects X 8 distances = 392) gives a rough estimation of the percentage of variation related to these different components. It revealed that approximately 39% of the variability in FD values is due to differences of scales (distance between sample points) while the rest (61%) is attributable to differences among transects at the same scale.

3.1 Whole-map versus individual transect estimation of fractal dimension

In this section, we examine the influence of using individual transects to estimate the fractal dimension of a variable distributed over a surface. As a standard of comparison, we used the fractal dimension obtained from a semivariogram derived from all the observation on the map. To eliminate the possible confounding effect of anisotropy (see section below), we confined our comparison to a single orientation (E-W axis). In the study area, whole-map FD values are generally lower than the mean FD values of the 49 transects although this tendency appears reversed at short distance of 0.3-km (Fig. 4). However, for each spatial scale taken individually, whole-map FD values are within the 95% confidence interval of the mean transect FD values. The important result here is that, although imprecise, fractal dimensions estimated from individual transects are unbiased. Clearly, the generality of this conclusion will need to be tested for other variables and in other landscapes. However, it suggests that it may not always be necessary to survey an entire area to estimate the fractal dimension of one of its components.

3.2 Influence of orientation

The variability in FD values attributable to the orientation of the transect is not negligible in this landscape (Fig. *5).* This difference is particularly striking between the SW-NE and SE-NW axes. At a spatial scale of 0.3-km, the fractal dimension of forest cover ranges from 1.53 in the SW-NE direction to **1.65** in the SE-NW axis. This confirms the sensitivity of the fractal dimension to the presence of anisotropy. In this region, the natural orientation of the woodlots appears to be in the SW-NE

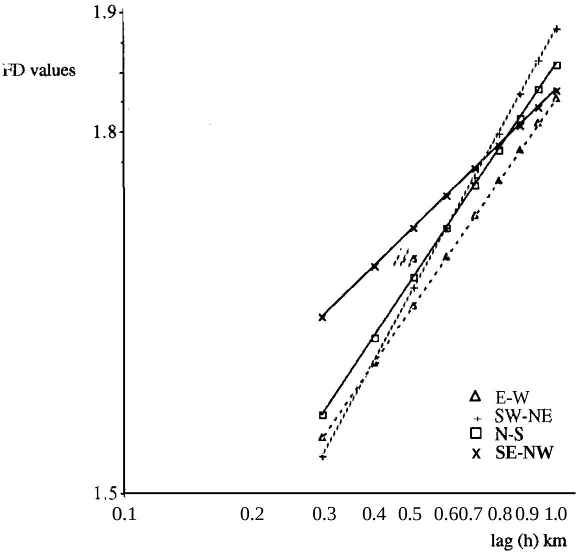

Fig. 5. Fractograms comparing different axes of variation in the forest cover. These fractograms result from the analysis of the entire map in which we have specified the computation of anisotropic semivariograms with a maximum band-width of **22.5".**

direction (Fig. 2). If we consider that most woodlots found in the region are on till deposits, the major axis of distribution of forest cover simply matched the major axis of till deposits (Bariteau 1986).

We found the highest rate of increase of FDs in the SW-NE axis and the lowest at an angle perpendicular to it and this, independent of spatial scale. This implies that, within its natural SW-NE axis, the woodlot distribution shows a spatial structure confined to a short scale range, *i.e.*, between **0.3** and l-km, which probably reflects the average size of woodlots or open areas. In the perpendicular SE-NW axis, woodlot distribution appears mostly dominated by large scale variations where the alternation between agricultural valleys and forested ridges appears to be the major source of variation. This may explain why, FD values in SW-NE axis are lower than in the SE-NW axis for short distances, and this is inverted for longer distances (Fig. 5). A two-way ANOVA on the orientation of FD values indicates that scale and orientation account for approximately 52% and 44% of the variability in fractal dimension, respectively.

3.3 *Grain size effect*

Change in sampling unit size influences several aspects of the semivariograms (Fig. 6). Both the nugget variance and sill (maximum semivariance) are slightly lower with the 25-ha sampling unit. The effect on the range is not as evident and appears to differ depending on the axis of investigation. Their combined effects may produce differences of approximately 10% in fractal dimension. In all cases, an increase in grain size produces a drop in the FD's, at least for the scale range analyzed. These differences are not always distributed regularly among all spatial scales (Fig. 6).

The effect of grain size appears related to the spatial arrangement of woodlots. Examining the landscape in a SW-NE axis (Fig. 6), we see that an increase in the grain size produces a slight drop in FDs for the large scales, indicating that a more regular pattern in forest cover variations at this scale is detected when we use a larger sampling unit. In contrast, in a SE-NW axis, an increase in the grain size led to a slight drop in the FD values for the short scales. We suggest that when a phenomenon is structured mostly on a short scale range (the SW-NE axis), the effect of increasing grain size is to structure the large scale variatons. In contrast, when a phenomenon is structured mostly by large scale variation, an increase in the grain size produces a structuring of their short scale changes.

Although little known, the effect of measurement scale on the different components of a semivariogram (*e.g.*, nugget variance, range, and sill) has been studied recently by Russo and Jury (1987) and Bramley and White (1991). Their results show that the nugget variance, range, and sill can be influenced to varying degrees by changes in grain size. Our results support these observations and lead us to conclude that the grain size at which a spatial pattern is quantified influences its characterization and, hence, measurements taken at different resolutions are not strictly comparable (Turner *et **al.*** 1989, Milne 1990).

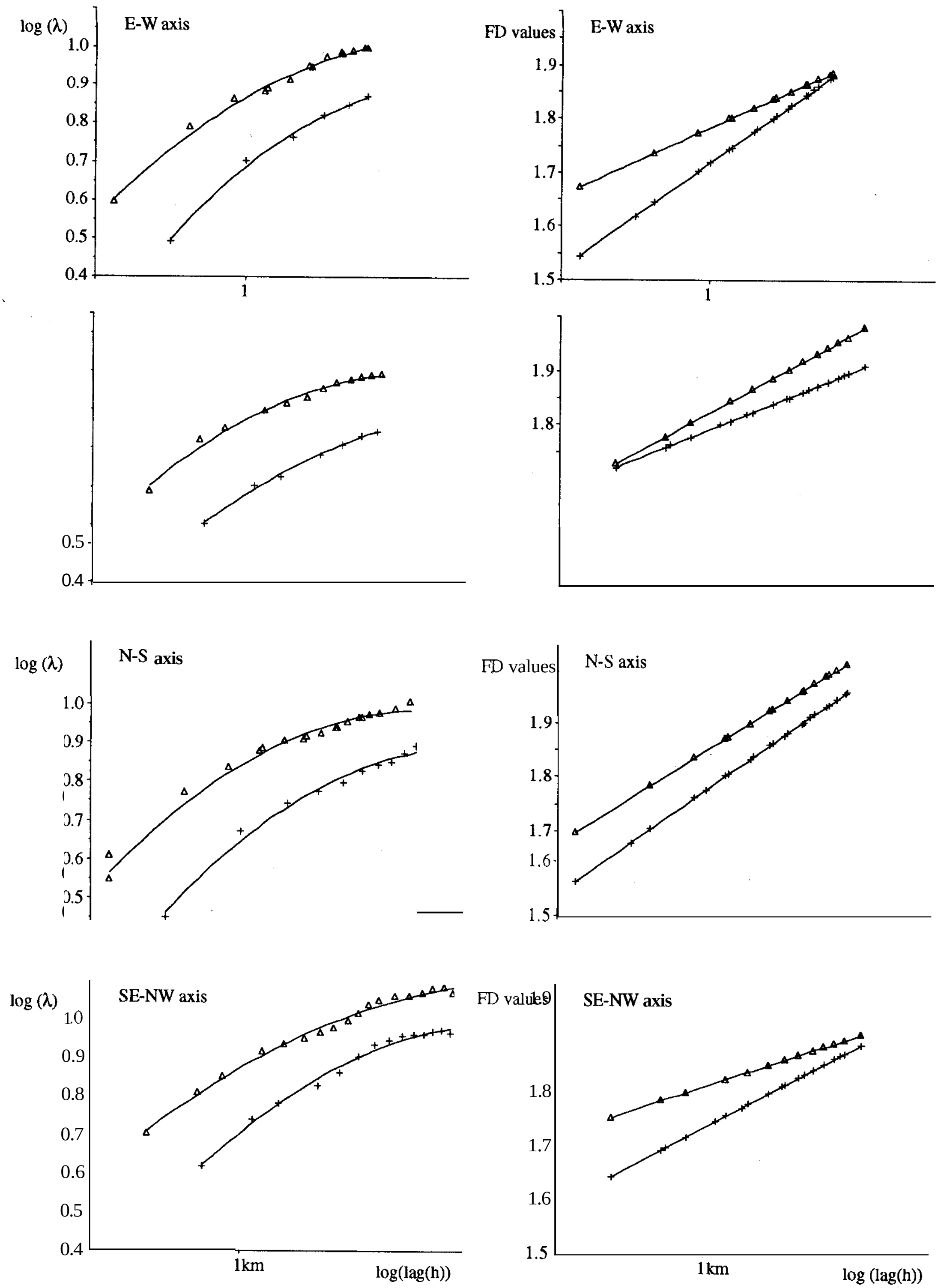

Fig. 6. Semivariograms and associated fractograms resulting from the analysis of the entire map using different axes. Triangular points denote values obtained from the analysis of the whole study area described by a grid 9-ha in cell size. Plus signs denote values obtained from the analysis of the same area described **by** a grid 25-ha in cell size.

4. Conclusion

The fractal dimension is often perceived as a measurement that is largely independent of scale. Scale effects, however, are present in many aspects of ecology, and are shown here to influence the estimation of FD values within a single landscape. Many other studies have also concluded that natural phenomena usually show a self-similarity only within a limited range of scales. In addition to this interesting lack of self-similarity, we have shown that several methodological factors confound and may often preclude accurate estimation and hence comparison of fractal dimensions. One often neglected aspect of scaling is the grain size of measured phenomenon. Our results show that the fractal estimation of a phenomenon can be influenced by the sampling unit size used to quantify it. Finally, scaling effects can be the result of the extent of a sample set, *i.e.*, the entire area covered by a sample set. From this point of view, we have seen that the use of transects can influence the estimation of the FD of a spatial pattern, as can the orientation of the transects, although it produces an unbiaised estimation of the FD.

Acknowledgments

We are thankful to R.W. Russell for his judicious comments in the review of this paper. We are also thankful to Mr. Stéphane Dyrda for assistance in digitizing the map. This research was supported by operating grants from the Natural Sciences and Engineering Research Council of Canada to Yves Prairie and Yves Bergeron.

References

Bariteau, L. 1986. La cartographie geomorphologique au 1:20 000 de modelés polygéniques: un exemple des basses terres du haut Saint-Laurent. Mémoire de maîtrise, Universite de Montreal, 185p.

Bramley, R.G. and White, R.E. 1991. An analysis of variability in the activity of nitrifiers in soil under pasture. II Some problems in the geostatistical analysis of biological soil properties. Aust. J. Soil Res. 29: 109–122.

Burgess, T.M. and Webster, R. 1981. Optimal interpolation and isarithmic mapping of soil properties. J. Soil Sci. 31: 315–331.

Burlando, B. 1990. The fractal dimension of taxonomic systems. J. Theor. Biol. 146: 99–114.

Burrough, P.A. 1981. Fractal dimension of landscapes and other environmental data. Nature 294: 240–242.

Burrough, P.A. 1983a. Multiscale sources of spatial variation in soil. I. Application of fractal concepts to nested levels of soil variation. J. Soil Sci. 34: 577–597.

Burrough, P.A. 1983b. Multiscale sources of spatial variation in soil. II. A non-brownian fractal model and its application in soil survey. J. Soil Sci. 34: 577–597.

Burrough, P.A. 1987. Spatial aspect of ecological data. *In* Data analysis in community and landscape ecology. Edited by R.H.G. Jongman, C.J.F. ter Braak and O.F.R. van Tongeren. Pudoc, Wageningen. p. 213–251.

Collins, S.L. and Glenn, S.M. 1990. A hierarchical analysis of species abundance patterns in grassland vegetation. Am. Nat. 135: 633–648.

Feldman, D.S. Jr., Hofmann, R., Gagnon, J. and Simpson, J. 1987. Statview II. Abacus Concept Inc. 278p.

Frontier, S. 1987. Application of fractal theory to ecology. *In* Developments in numerical ecology. Edited by P. Legendre and L. Legendre. Springer-Verlag, New York, pp. 335–378.

Journel, A.G. and Huijbregts, C. 1978. Mining geostatistics. Academic Press, London. 600 pp.

Krummel, J.P., Gardner, R.H., Sugihara, G., O'Neill, R.V. and Coleman, P.R. 1987. Landscape patterns in disturbed environment. Oikos 48: 321–324.

Logan, B.E. and Wilkinson, D.B. 1990. Fractal geometry of marine snow and other biological aggregates. Limnol. Oceanogr. 35: 130–136.

Mandelbrot, B.B. 1982. The fractal geometry of nature. Freeman, San Francisco.

Milne, B.T. 1990. Lessons from applying fractal models to landscape pattern. *In* Quantitative methods in landscape ecology. Edited by M.G. Turner and R.H. Gardner. Springer-Verlag, New York, pp. 199–235.

Palmer, M.W. 1988. Fractal geometry: a tool for describing spatial patterns of plant communities. Vegetatio 75: 91–102.

Robertson, G.P. 1987. Geostatistics in ecology: interpolating with known variance. Ecol. 68: 744–748.

Russo, D. and Jury, W.A. 1987. A theoretical study of the estimation of the correlation scale in spatially variables field. 1. Stationary fields. Water Resources Research 23: 1257–1268.

Sugihara, G. and May, R.M. 1990. Application of fractals in ecology. TREE 5: 79–86.

Trangmar, B.B., Yost, R.S. and Uehara, G. 1985. Application of geostatistics to spatial studies of soil properties. Adv. Agron. 38: 45–94.

Turner, M.G., O'Neill, R.V., Gardner, R.H. and Milne, B.T. 1989. Effect of changing spatial scale on the analysis of landscape pattern. Land. Ecol. 3: 153–162.