

Forest spread and phase transitions at forest-prairie ecotones in Kansas, U.S.A.

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Abstract

The spread of gallery forest habitat into upland areas is of substantial interest to resource managers because such spread has many implications for the management of grassland and forest habitats. This study used a dynamic percolation model to examine the potential rates of spread or invasion of forest in eastern Kansas. Aerial photos taken 16 years apart at the Fort Riley training base were used to calibrate a spatially explicit contagion model of forest spread to interpolate and extrapolate the forest spread processes. Results fit the actual pattern of spread well, as measured by both visual inspection and a multiscale fractal measure of pattern. Comparisons to a long-term fire-exclusion experiment in Geary County, Kansas, and to the Konza Prairie also provided validation. Both the simulation and the 100-year Geary County series showed an interesting pattern of forest spread. Spread was slow and steady until about 20% forest cover was reached, at which point the rate increased. We conclude that this self-accelerating response is due to spatial patterns created by the spreading forest that tend to accelerate the growth process after a critical point is reached. On the basis of theoretical study and experimental simulation of the percolation phase transitions, we suggest that fractal dimensions in a transient ecotone of binary mixtures (*e.g.*, trees and grasses) should range between 1.56 and 1.8958, and the critical fractal dimension during ecotonal phase transitions should be 1.7951. This critical point of about 18.5% forest cover that we predicted was close to the observed result and might represent a phase transition at the forest-prairie ecotone.

1. Introduction

Riparian or gallery forest is highly productive, diverse in species, and important for wildlife. As documented by the original land-office survey of Konza Prairie in 1858, these forests have always been restricted to narrow strips along natural springs and seepages, creeks, and prairie streams (Abrams 1986a). The tallgrass prairie dominated the landscape, aided by periodic drought and natural fires ignited by lightning strikes or set by Native Americans to herd or attract game. With European settlement came fire control and an expansion of gallery forest up the sides of valleys, indicating that fire, rather than dry conditions, controlled the growth of woody vegetation (Abrams 1986a). This expansion continued into this century, as docu-

mented by an increase in forest area on Konza Prairie from 111 to 206 ha from 1939 to 1978 (Abrams 1986a,b). Knight *et al.* (1994) estimated an increase from 157 ha in 1939 to more than 241 ha in 1985.

For the purposes of conservation and habitat management of grassland and riparian forest, the forces controlling the positions of the gallery forest-prairie boundaries are of interest; for example, military training would be enhanced by expansion of forest area at the Fort Riley military base, according to base staff. Various wildlife species would also benefit from increased forest area, though, of course, others would be hurt. Of interest is whether natural rates of spread are sufficient or if they can be enhanced to promote increased forest area within reasonable time frames. To further our

understanding of dynamic processes of the forest-prairie ecotone system, we used dynamic percolation theory and a simulation model to examine (1) the potential rates of spread or invasion of forest into tallgrass prairie and (2) the possibility of phase transitions at forest-prairie ecotones in eastern Kansas. A percolation-based phase transition framework for analysis of ecotonal dynamics has been introduced in this research. The ecotonal tension zones under study here provide a particularly good situation for the study of ecological phase transitions. These ecotones can be simplified to the situation of binary mixture phases (or a two-phase vegetation pattern): trees (forest) and grasses (tallgrass prairie). The typical boundary or phase transition between grassland and forest in Kansas is somewhat of a puzzle. One might expect the boundary change to be either more gradual or catastrophic, or both, because abiotic factors do not vary in such a discontinuous manner, but some others (*e.g.*, fire) do. Gosz and Sharpe (1989) proposed the existence of an instability at such an ecotonal transition, such that grassland areas and forest areas are each self-amplifying, stable configurations. This proposal leads to abrupt transitions between the types and sharp ecotonal boundaries. Recently, Milne *et al.* (1995) used the concept of critical density in percolation phase transition theory to study changes of piñon-juniper woodland ecotones. In this study, we propose an alternative and more objective and general way to examine changes over time in forest-prairie ecotonal boundaries in terms of this phase transition framework. The ideas and principles developed for analysis of changes at the ecotones could also be considered for the analysis and detection of potentially abrupt physiognomic changes through time (thresholds) over large regions.

2. Theoretical consideration of ecotonal phase transitions

Phase transitions have interdisciplinary features. The concept of phase transition that has been traditionally related to the physics of condensed matter nowadays spreads over various branches of science and engineering (Kotecky 1993). Under certain conditions, a number of systems in nature

undergo phase transitions; for example, a gas-liquid-solid diagram, magnetic ordering, the Belousov-Zhabotinskii chemical reaction, chaos, superconductivity, metal-dielectric transition, and crystallization (Zallen 1983; Uzonov 1993). The theory of phase transitions and critical phenomena has been considered as an independent scientific theory (Yukalov and Shumovsky 1990).

The study of phase transitions in ecology has included the applications of catastrophe theory to pest outbreaks (Ludwig *et al.* 1978; Casti 1982) and to forest ecosystem exploitation (Gatto and Rinaldi 1987; Loehle 1989). Other approaches to phase transitions include applications of percolation theory to landscape ecology (Gardner *et al.* 1987; Milne *et al.* 1995), forest fire (McKay and Jan 1984), and epidemic spread (Grassberger 1983, 1985); and application of nonlinear stability theory to nonequilibrium phase transitions of vegetation landscapes in southern Texas savanna (Li 1995a). These studies indicated that developing an ecological phase transition theory might be useful for understanding ecosystem dynamics. In particular, this research may contribute to a determination of ecotonal changes because ecotones are transitional areas between adjacent ecological systems or, more traditionally, between types of vegetation. Ecotones occur under two types of conditions: (1) steep gradients in physical environmental variables that directly affect key ecological processes and the distribution of organisms; and (2) threshold or nonlinear responses to gradual gradients in the physical environment that cause large changes in ecosystem dynamics and the distribution of dominant species (Gosz 1992; Risser 1995). Identifying the thresholds of the phase transitions at an ecotone is important for characterizing the ecotonal dynamics.

A phase state (or, simply, a phase) is a state of a macroscopic system that is qualitatively different in its characteristics from other states of the same system. A phase transition is a qualitative change in the state of a system under a continuous infinitesimal change in external parameters (Uzunov 1993). Simply, we may also say that the phase transition is a transition from one phase state of a system to another. Vegetation, for example, is often described in physiognomic terms by the dominant life-form: trees (forest), shrubs (shrub land), grasses (grassland), and combinations of these (woodland; savan-

na). Types of vegetation can be considered ecological phases, and transformations from one type to another can be considered ecological phase transitions. A system that is poised between two or more phases is an ecotone.

Critical phenomena are peculiarities of the behavior of a system in the vicinity of a phase transition. An equilibrium phase transition is that in which the state of a system above and below the transition point is at equilibrium and stable. For example, after a substance has crystallized from a supersaturated solution, both the crystal and the solution are at energetic equilibrium, and the boundary does not change with time. However, nonequilibrium or dynamic phase transitions occur in which a qualitative change in the state of a system comes from a continuous change in the external parameter, time. In this case, naturally, nothing can be said about the thermal stability of states before and after the phase transition. These nonequilibrium phase transitions are associated with dynamic processes of self-organization in systems with nonzero energy fluxes and are studied by the so-called synergetics (Yukalov and Shumovsky 1990). An example would be the border of a spreading fire in a landscape; this border is not stable and no equilibrium exists. Most ecotonal systems are in a state far from thermal equilibrium (Li 1995a).

Phase transitions are characterized by a fundamental change in an order parameter (Uzunov 1993). They can be discontinuous (first-order phase transitions) and continuous (second-order phase transition). The point of phase transitions is usually called the critical point. Strictly speaking, it is not necessary at all that a transition be realized just at a point; for example, an ecotone can be considered a phase transitional area or zone on [a, b].

The traditional models for phase transitions are thermodynamic models. These models successfully describe the essential experimental observations in most systems in physics and chemistry; however, recent studies have typically observed fractal patterns in systems that develop far from equilibrium during phase transitions, especially where kinetic growth processes are active (Schaefer *et al.* 1990; Stinchcombe 1990). Fractal analysis has become a powerful tool in phase transition studies (Sahimi 1994). These observations may also provide a great

opportunity for us to link fractal analysis and spatial phase transitions together in ecotone studies because of the difficulty of parameterizing thermodynamic models. For most establishing vegetation, in fact, initial growth must take place far from equilibrium, exactly the conditions where fractal correlations are expected (Li 1995b). In this study, we used the simplest phase-transition model in modern theories of phase transitions, the percolation model, together with fractal analysis, to study ecotonal dynamics.

Percolation processes were first developed by Flory (1941) and Stockmayer (1943) to describe how small branching molecules react and form very large macromolecules. This process is very similar to expansion of forest or shrubs in grassland (Li *et al.* 1992). In the mathematical literature, percolation was introduced by Broadbent and Hammersley (1957). They originally dealt with the concept of the spread of hypothetical fluid particles through a random medium. The terms fluid and medium were viewed as totally general: a fluid can be a liquid, vapour, heat flux, infection, fire or forest spreading, and so on. The medium where the fluid is carried can be the pore space of the subsurface, an array of trees, grassland, or the universe. Percolation provides an intuitively appealing and transparent model for dealing with the unruly geometries that occur in many random media. The percolation threshold is a prototype phase transition that occurs as one varies the richness of interconnections present (a generalized density or composition). For a detailed introduction to percolation models and theory, see Stauffer (1985) and Sahimi (1994). In ecology, percolation models have been applied to the spread of epidemics (Mollison 1977, 1986; Kuulasmaa 1982; Grassberger 1983, 1985; Kuulasmaa and Zachary 1984; Cardy and Grassberger 1985; Mollison and Kuulasmaa 1985; Cox and Durrett 1988), forest fire (McKay and Jan 1984; Ohtsuki and Keyes 1986; von Niessen and Blumen 1986), microbial transport (Li *et al.* 1995), and ecotone and landscape ecology (Gardner *et al.* 1987; Milne *et al.* 1995).

Milne *et al.* (1995) used critical densities or percolation thresholds of percolation models to identify woodland ecotonal phase transitions. In percolation theory, the numerical value of every percolation quantity for any percolation probability p de-

depends on the microscopic details of the system, such as its configuration or neighborhood; however, near the bond or site percolation threshold p_c , most percolation quantities obey scaling laws that are largely insensitive to the network structure and its microscopic details. Nevertheless, the percolation threshold p , still varies with configuration or neighborhood. Percolation theory tells us that the topological exponents, including fractal dimension near p , are completely universal (Stauffer 1985; Sahimi 1994). These exponents are independent of the microscopic details of the system, and only depend on the dimensionality of the system. This discovery will allow us to use fractal dimensions directly to detect spatial phase transitions of an ecotone, regardless of the spatial configuration of the object of concern.

On the basis of theory and computer simulation of general random percolation transitions (Stauffer 1985; Stinchcombe 1990; Sahimi 1994), the relation between critical exponents and a dimension of a fractal occurring in percolation is

$$Fd = d - \beta/\nu \quad (1)$$

where d is the Euclidean dimension, β and ν are the critical exponents, and $(-\beta/\nu)$ can be regarded as the length-scaling exponent. On the basis of currently accepted values of critical exponents (Stauffer 1985; Sahimi 1994), we have fractal dimensions, $FD(p = p_c) = 1.8958$, $FD(p > p_c) = 2$, and $FD(p < p_c) = 1.56$. Here let us consider the forest spread as a dynamic percolation process. The percent cover is the percolation probability. When forest cover is below the percolation threshold p_c , the landscape is tallgrass prairie dominant. When forest cover is above p_c , the forest can spread through the entire landscape, and the landscape becomes forest dominant. On the basis of this intuitive argument, we suggest that fractal dimensions of forest spatial distributions can be used to characterize ecotones in landscapes and should range between 1.56 and 1.8958 at an ecotone. This result can also be used to monitor ecotonal movement and responses to climate change.

Spatial phase transitions of an ecotone between tallgrass prairie and forest are complex. In general, if environmental or management conditions change in ways that are beneficial for one of the adjacent ecosystems, patch size is likely to increase in this

system, and that ecosystem is likely to invade habitats previously unsuitable in the adjacent system (Risser 1995). Let us assume 70% driving forces (or controlling factors and constraints) are working toward a phase transition of forest spreading in this ecotone. We could simply calculate a new fractal dimension of this critical transition in the two-dimensional lattice as

$$FD_{70\%} = 1.8958 - (1.8958 - 1.56) \times (1 - 0.7) = 1.7951. \quad (2)$$

If there are 80% driving forces, we have $FD_{80\%} = 1.8286$. We could use the critical fractal dimension to detect a phase transition at the forest-tallgrass prairie ecotone. Now we can state our hypothesis in this study, that is, if the fractal dimension of forest cover is above 1.7951 or 1.8286, the forest invasion at forest-prairie ecotone may spread to the entire system. We will test this hypothesis on the basis of actual and simulated data at Fort Riley.

Computer simulations of invasion percolation with trapping by Wilkinson and Willemsen (1983) in two dimensions indicated that the fractal dimension of the invader cluster is about 1.82. The experiments of Lenormand and Zircon (1985) supported this result that $1.80 < FD < 1.83$. This fractal dimension is very close to our suggested one.

Percolation theory can tell us whether a system is macroscopically connected or not. This macroscopic connectivity is of fundamental importance to many phenomena involving random media (Sahimi 1994). Moreover, universal scaling laws near the percolation threshold tell us which aspects of a given dynamic system are important in determining its macroscopic properties and which aspects are not relevant, and therefore we do not have to worry about them to collect data and build very complex models.

3. Description of study

A very useful, although unintended, natural experiment was conducted on the spread of forest in eastern Kansas at the Fort Riley Army training installation. In 1965, the installation purchased several large tracts of land to expand its training capability. Before its purchase by the government for military training activities, the land, with several riparian

stream valleys extending across it, was used predominantly as cropland and pastureland. After the purchase, agricultural activities were halted on most of the area, eliminating the plowing of forest edges, the grazing of cattle, and the cutting of timber. No effort was made to replant trees. Management consisted largely of prescribed burning to improve the condition of the range. Wildfires occasionally resulted from both lightning strikes and training activities. According to the Fort Riley resource management staff, while fires were not used specifically to eliminate forest or to prevent the spread of forest, neither were forests particularly protected. Increased moisture and natural firebreaks near streams appeared to provide some protection from fires, thus preventing severe damage to gallery forest zones. Even after the area was converted from agriculture to grassland by the Army installation, the network of roads and the streams likely acted as effective firebreaks. Controlled burns by base staff are typically conducted under conditions that minimize the risk of escape of fire. The net result of these factors is probably a much lower frequency and intensity of fire in this area than before settlement. We particularly noted rapid encroachment of brush in small areas bounded by the intersections of roads and streams. These areas have effective firebreaks and are too small for controlled burns. This observation was an indication that controlled burning is conducted at the base at a coarse scale; not every little area is burned.

To evaluate the change in forest area, aerial photographs from 1969 and 1985 were compared with regard to vegetation class and cover, with the period of recovery beginning in 1965, when the land was purchased. These photographs show a number of abandoned farm fields with windbreaks along section lines. Within this landscape, several distinct, wooded stream valleys can be seen, along with a number of hard-surfaced roads and staging areas. Three sites differing in topography were chosen for comparison: site 1, with 850.5 ha, to the west and adjoining the central firing range; site 2, with 510 ha, on the opposite side of the firing range; and site 3, with much flatter topography, at the western edge of the base. Forest area was visually delineated and manually drawn on U.S. Geological Survey 7.5-minute topographic sheets for

the 1969 photographs. On these same maps, areas of increased forest in the 1985 photographs were added. In no case could any local decrease in forest area be observed, supporting the contention that disturbances caused by fire and by training were not sufficiently intense to cause a recession of the forest area. The hand-drawn forest area maps were digitized and entered into a GRASS geographic information system (GIS) database.

A simple model of forest spread was constructed to simulate these data for the purpose of interpolation and extrapolation. The spread model was based on the concept of contagion, one kind of percolation modeling approach (Mollison 1977; Cox and Durrett 1988). In this approach, forest only spreads at its edges and not by wide dispersal of seeds into open areas. Similar models have been applied to fire spread. The first step in modeling was to evaluate the contagion assumption. Field observations clearly show brush spreading out gradually from the forest edge in various places. The biological basis for this mode of spread is threefold. First, tree seedlings have difficulty invading prairie sod because of the thick layer of surface roots. Trees shade their margins somewhat, thus suppressing grasses sufficiently to allow tree seedlings to become established. In addition, tree species provide for a zone of higher moisture, encouraging the growth and spread of cool-season grasses, which are more difficult to burn. This factor and the limitations on dispersal (Kucera 1960) control rates of spread. Second, mycorrhizae play an important role (R.M. Miller, Argonne National Laboratory, Argonne, Ill., personal communication). *Oaks* and many other species in Kansas forests are ectomycorrhizal, but ectomycorrhizae grow only in association with forest species and not with prairie species, which are largely endomycorrhizal. The spores of ectomycorrhizal fungi do not disperse widely, and their requirements for leaf litter as growing media, plus more acidic soil and higher litter quality, restrict them to the forest. The woody species (red cedar, sumac, and *Prunus* sp.) that invade the grassland more easily are endomycorrhizal (R.M. Miller, Argonne National Laboratory, Argonne, Ill., personal communication). Third is the influence of periodic fires. The sod formation of tall grasses creates a continuous carpet that allows fire to travel at tremendous speeds. These fac-

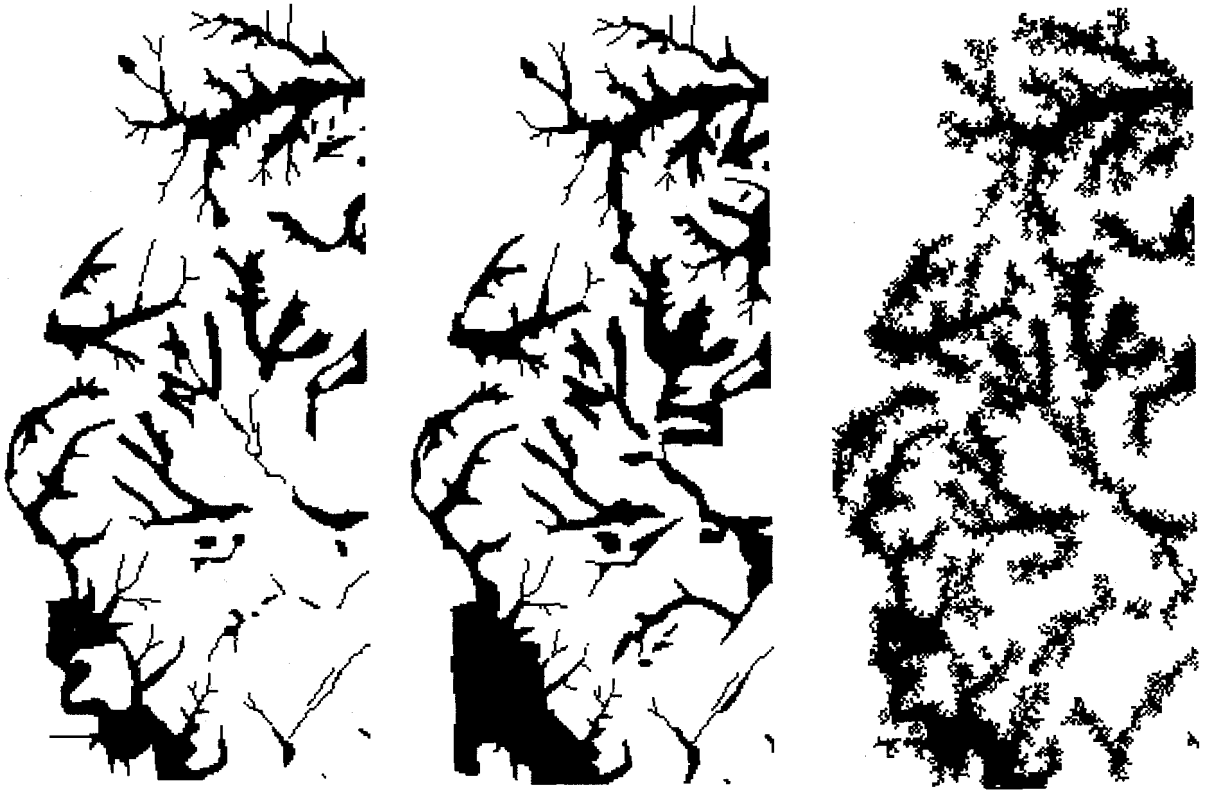


Fig. 1. Forest coverage in area 2 for (a) 1969 (actual data), (b) 1985 (actual data), and (c) 1985 (simulation).

tors can drive a fire deep into the fringes of a forested area, eating away at the margins of the forest over a period of years and several fires. Only rocky outcroppings, natural springs with the surrounding moister soils, and riparian areas with dense thickets of vegetation provide any protection against such a force. Because individual trees that become established in the prairie are often killed by fire, lightning strikes, and drought, the spread of forest into grassland in this system is largely confined to the edges of existing forest.

We modeled this process of spread with a contagion model. A cell (10 m wide) was modeled to become forest during a one-year interval with some probability q if the cell was adjacent to existing forest but was modeled to remain grassland otherwise. The parameter q was estimated by iterative approximation. On each iteration, the contagion model was run for 16 years, with the initial condition given by the 1969 photograph and with q given by the estimate at that iteration. Total forest acreage at the end of the period was compared with

that estimated from the aerial photograph for 1985 to determine the convergence of q .

4. Results and discussion

Several results are immediately apparent. On sites 1 and 2, forest increased by 72% and 42%, respectively, over the 16-year period. Virtually all new forest area was contiguous with existing forest; however, site 3 had no detectable change in forest area. Site 3 has far fewer streams and roads to act as firebreaks and has a flatter topography than do the other two sites, and the forest consists of very narrow strips along streams. Most likely, fire has been effective in preventing the spread of forest on site 3.

In the simulation model, the estimate of q obtained was 0.04. Because growth was modeled as a stochastic process, the exact pattern of forest growth was not simulated. Visual and statistical measures were used to evaluate the goodness of fit

Table 1. Actual and simulated changes in woody cover at Fort Riley.

Area	Actual 1969 area (ha)	Actual 1985 area (ha)	Simulated 1985 area (ha)
1	6,991	12,025	9,578
2	9,050	12,834	12,925

of the model. Visually, the match between actual and projected forest areas for site 2 is quite good (Fig. 1). As measured by area, simulated area on the area 2 site matches the actual area almost exactly (Table 1). The simulation for area 1 underestimated the actual 1985 results, but this difference can be explained as a bias in the data for area 1, with the simulated area matching other results closely (see below).

A test of the model was performed on the basis of an independent data set. Bragg and Hulbert (1976) evaluated forest area over a 100-year period on a site in Geary County, Kansas, only a few miles from Fort Riley and the Konza Prairie. On the Geary County site, a large tract of land from which fire had been excluded for more than 100 years was compared with an adjacent tract where fire was not excluded. On the basis of section line and aerial photographic data, forest spread by only a few percent over the interval on the burned tract, but forest coverage increased from less than 5% to nearly 60% in areal extent on the unburned tract. We used this longer term record as a check on our contagion model.

In the data from Geary County, the 1856 value was estimated from survey records; values for 1937, 1950, 1956, and 1969 were estimated from aerial photographs. In the absence of other information, we assumed a linear trend from 1856 to 1937 (Fig. 2). Changes over time from 1939 to 1985 for areas 1 and 2 at Fort Riley and for the Konza Prairie were superimposed on this plot by placing the beginning year for each line on the year in which the forest cover matched that in Geary County. In this way, the behavior as a function of extant cover could be compared directly. Plotted in this way, the spread rate for Konza Prairie falls below the Geary County trend, probably because of the intense controlled burning at Konza Prairie. This observation compares to almost no increase in woody cover at the Geary County fire site over

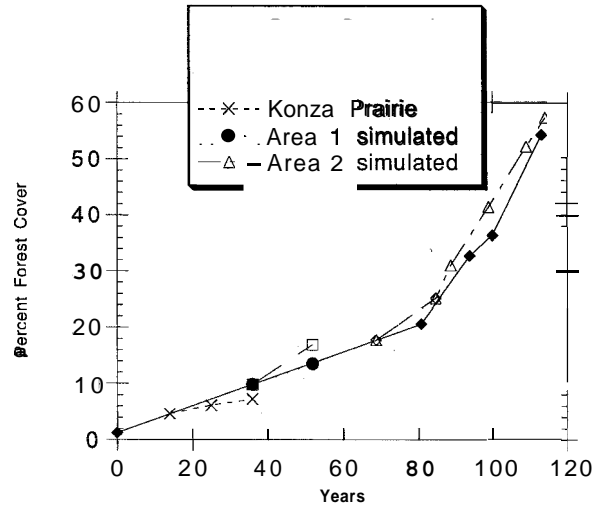


Fig. 2. Long-term trends in forest area after fire suppression. Data of Bragg and Hulbert (1976) for Geary County and of Knight *et al.* (1994) for Konza Prairie are compared with simulated spread. To test for similar trends with time, data from Fort Riley and Konza Prairie are overlaid on the year at which their initial percent cover matches that in Geary County. This procedure allows rates of spread in an area to be compared with the initial year in which Geary County forest cover matches the particular site.

more than 100 years. We may safely infer that the fire regime at Geary County was more intense than at Konza Prairie, because at Konza, small areas are burned in a controlled manner at fixed intervals. Thus the intermediate rate of spread at Konza fits these trends as a function of fire intensity. The trend for forest extent in area 2 follows the Geary County fire trend closely, and the increase to 12,834 ha after 16 years is almost exactly simulated at 12,925 ha. This close match indicates that forest in area 2 is increasing at a rate comparable to the rate of forest spread where fire is absent; this rapid spread probably results from the effect of firebreaks due to roads and the more dissected topography on this site. However, in area 1, the measured increase exceeds that observed at Geary County. Because the simulated spread for area 1 falls on the Geary County trend line, we conclude that we either underestimated the area 1 forest cover in 1969 or overestimated it in 1985, or both. Large tracts in area 1 appeared somewhat shrubby in 1969 and more shrubby in 1985 (but were not solidly wooded), causing considerable ambiguity about classifying regions on the photographs as

woody in each of the years. This ambiguity probably led to the underestimate-overestimate problem.

Finally, a longer simulation was conducted for area 2 as a test. Area 2 was chosen because of the good fit between both actual and simulated changes for the 16-year period and the Geary County data. The 1969 map was simulated without fire for 45 years and was overlaid on the Geary County plot, as before. The result is a close match to the Geary County trend line, including the increase in the rate of spread in the later years.

These results have several implications. The shift in the rate of spread to a faster rate in later years (giving the inflection point in Fig. 2) could give rise to a climate-change interpretation, with conditions after 1939 being more favorable to forest; however, our stochastic spread model is stationary with respect to climate, and constant parameters yield the same rising curve, including the inflection point. Therefore, we must conclude that the increased rate of spread in later years is a function of topography and the manner in which fingers of invading forest increase the edge at which forest can spread farther, thus increasing the rate. This self-accelerating process begins at about 20% forest cover and is not related to climate change. We also note that the forest areas studied spread at a rate equivalent to that for the no-fire site in Geary County. We thus suspect (on the basis of road and stream locations) that roads and topography somewhat protect these two areas from fires. As noted, the unprotected site 3 showed no spread and thus matches the behavior of the Geary County fire site.

A final evaluation of this simulation model involves the ability to match the spatial pattern of newly forested areas across scales, as well as the total area. A widely used measure of pattern is the fractal measure (Loehle and Wein 1994), although other methods are also available (Li *et al.* 1993); for example, Rex and Malanson (1990) showed consistent changes in the fractal dimension of remnant forest in Iowa as the remnants became progressively confined to stream valleys by agricultural conversion. Knight *et al.* (1994) estimated fractal dimensions of forest patch shapes in spreading forest areas at Konza Prairie. Both studies used the area-perimeter relationship to calculate fractal dimensions; however, this method of estimating frac-

tal dimension only provided the information of complexity change of average patch shapes (Li 1995b) and might not reflect the change of expansion of gallery forest. It is therefore not a surprise that these studies concluded that no statistically significant differences existed between the fractal dimensions over time.

We use a multiscale information fractal model (Loehle and Wein 1994; Loehle and Li 1995) to test the changes of the spatial patterns over scales or resolutions. We begin by assigning to each pixel a 0 or 1 on the basis of whether it is nonforested or forested. Each value is divided by the total number of forested cells N to get a P value for that cell. With a change in scale, boxes of different sizes have changing P values on the basis of how filled the boxes are (number of filled cells within a given box size, divided by N). We compute the estimate d_f of the information fractal dimension discretely at a series of scales by using

$$d_f(\epsilon) = \frac{\sum_{i=1}^{K(\epsilon)} P_i(\epsilon) \log P_i(\epsilon) - \sum_{j=1}^{K(\gamma)} P_j(\gamma) \log P_j(\gamma)}{\log [1/\epsilon] - \log [1/\gamma]}, \quad (3)$$

where $K(\epsilon)$ is the number of boxes of width ϵ , and γ is the next larger box size to ϵ . Equation 3 computes d_f as the slope of the line, at each scale, given by the information measure versus $\log (1/\epsilon)$. For computational details, see Loehle and Wein (1994). For detailed statistical properties of this fractal dimension, see Loehle and Li (1995).

We applied this index at a series of scales to the historical and simulated data from Fort Riley. The multiscale profile (Fig. 3) shows that at fine scales of resolution (left side), linear features dominated in 1969 (d_f close to 1). At coarser scales, the dimension approaches 2 as we encompass larger patches and uniformly spaced small fingers of forest (see Fig. 1). At still coarser scales, we again pick up a decrease in d_f as we sample the irregular distribution of entire forested valleys. For 1985, the entire left-hand portion of the d_f curve is shifted up, representing a thickening of the entire network of valley forests. The simulated 1985 data match this shift quite well at almost all scales, indicating a good ability to capture changes in pattern with this model.

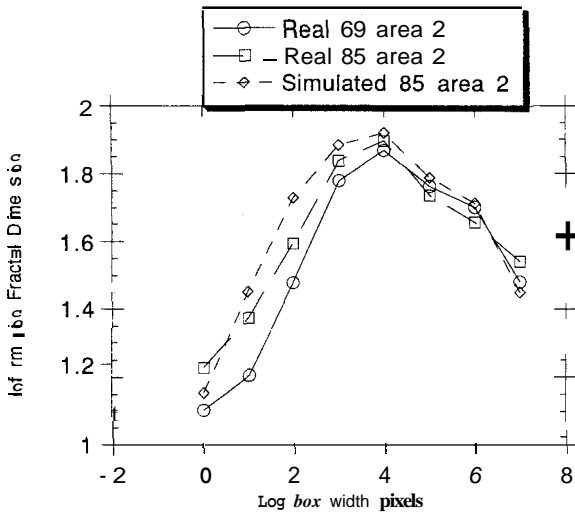


Fig. 3. Fractal profiles for real and simulated data for area 2 for 1969 and 1985. Scale is log of the box width in pixels, with pixels being 10 m on a side.

The results of this study have implications for the study of ecotones in the framework of spatial phase transitions. A phase transition is a fundamental change in phase, organization, or structure that occurs at a critical value of some controlling factor, such as density. Several studies have hypothesized that sharp ecotonal boundaries could result from phase transitions in space of the ecosystem structure (Gosz and Sharpe 1989; O'Neill *et al.* 1989; Kay and Schneider 1992). In particular, Milne *et al.* (1995) have applied a percolation modeling approach to the identification of critical densities of ecotones as phase transitions. The acceleration in the rate of spread that occurs at about 20% forest cover (Fig. 2) suggests a phase transition. At this critical density, a sudden step increase occurs in the rate of spread. This observation suggests that a true phase transition may exist at the forest-prairie ecotone.

In order to theoretically verify this observation and test the hypothesis that we proposed earlier, we compare this value with the theoretical prediction of percolation phase transition. On the basis of the ecological phase transition framework that we introduced earlier, we need to calculate global information fractal dimensions of actual forest cover in 1969 and 1985 in area 1 and 2. We used a simple linear regression relationship to estimate this information fractal dimension (Li 1995b):

Table 2. Fractal dimensions of actual changes in woody cover at Fort Riley.

Area	Year	Woody cover (%)	Information fractal dimension
1	1969	9.8	1.7609
	1985	16.8	1.7895
2	1969	17.75	1.7898
	1985	25.17	1.8201

$$H(\epsilon) = H(0) - FD \log \epsilon \quad (4)$$

where $H(\epsilon) = -\sum P_i(\epsilon) \log P_i(\epsilon)$ at scale ϵ , and FD is the information fractal. The scales that we used 1–5 log box width pixels (for results, see Table 2). Using actual and simulated data, we found a best-fit linear relationship region between the fractal dimensions and the forest cover near the phase transition zone (10%–30%):

$$FD = 1.7228 + 0.3904 (\text{Forest Cover } \%), \quad R = 0.996, P < 0.001 \quad (5)$$

If we used the critical fractal dimension $FD_{70\%} = 1.7951$, we could derive that the critical forest cover should be 18.5% as a phase transition from equation 5. If we used $FD_{80\%} = 1.8286$, the critical forest cover is 27.1%. This result of 18.5% forest cover is very close to the observed 20% from Fig. 2. Such a conclusion creates a direct link with the work of Milne *et al.* (1995), which focused on the identification of ecotones and forest clusters with percolation techniques; however, our method presents a simpler objective and general way to identify phase transitions at an ecotone.

4. Conclusions

Analysis of the change in forest in eastern Kansas indicated that gallery forest can spread and fill stream valleys relatively rapidly, even in the absence of human intervention like tree plantings. Forest spread to uplands is probably limited by fire except on scattered sites where the soil is too thin to support trees. The spread of gallery forest is not incompatible with controlled burning of grasslands when some topographic protection of the forest exists. Thus, an increase in gallery forest habitat in

this type of topography is feasible. An extensive area of upland forest is possible in 30 to 60 years if fire is controlled. This report shows that a contagion model of forest spread can match both changes in spatial pattern and long-term changes in forest area, both with and without fire. On the basis of theoretical and experimental studies of percolation phase transitions, we suggested that the fractal dimensions in a transient ecotone with a two-phase vegetation pattern (*e.g.*, trees and tallgrasses in our case) should range between 1.56 and 1.896, and the critical fractal dimension during spatial phase transitions in an ecotone should be 1.7951. Because these fractal measurements at the critical boundary are independent of the two-dimensional lattice structure and are easily calculated, they can be used as thresholds or nonlinear responses in ecotonal dynamics. This case that we have calculated is very close to field observations and may provide an example of a phase transition at an ecotone. Our method presented here may also provide reproducible, objective, and simple applications in other ecotone and landscape studies.

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